GCE MARKING SCHEME

SUMMER 2016

Mathematics – M2
0981/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE Mathematics - M2

Summer 2016 Mark Scheme

Q	Solution	Mark	Notes
1(a).	$x = \int 12t^2 - 7kt + 1\,\mathrm{d}t$	M1	At least one power increased
	$x = 4t^3 - \frac{7k}{2}t^2 + t + (C)$	A1	correct integration
	$t = 0$, $x = 3$	m1	use of initial conditions
	$C = 3$		
	$x = 4t^3 - \frac{7k}{2}t^2 + t + 3$		
	$t = 2$, $x = 16$	m1	values substituted
	$16 = 32 - 14k + 2 + 3$		
	$k = \frac{3}{2}$	A1	cao
1(b).	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	M1	At least one power decreased
	$a = 24t - 10.5$	A1	correct differentiation ft k. accept k
	$F = 4(24t - 10.5)$	m1	4xa
	When $t = 5$		
	$F = 4(24\times5 - 10.5)$		
	$F =$ <u>438 (N)</u>	A1	ft k. –ve values A0

Q	Solution	Mark	Notes
2(a)	$u_H = 24.5\cos30° = (12.25\sqrt{3})$	B1	
	$u_V = 24.5\sin30° = (12.25)$	B1	
	$s = ut + 0.5at^2$, s=0, u=12.25, a=(±)9.8	M1	oe complete method
	$0 = 12.25t - 0.5\times9.8\times t^2$	A1	
	$t = \dfrac{12\cdot25}{4\cdot9}$		
	$t = 2.5$	A1	
	Range = $2.5 \times 12.25\sqrt{3}$		
	Range = <u>53.04 (m)</u>	A1	cao
2(b)	$v^2 = u^2 + 2as$, v=0, u=12.25, a=(±)9.8	M1	oe complete method
	$0 = 12.25^2 - 2\times9.8\times s$	A1	ft u_V
	s = <u>7.65625</u> = <u>7.66 (m)</u>	A1	answers rounding to 7.7 ISW
2(c)	Required speed is 24.5 ms^{-1} downwards at an angle of 30° to the horizontal.	B1	

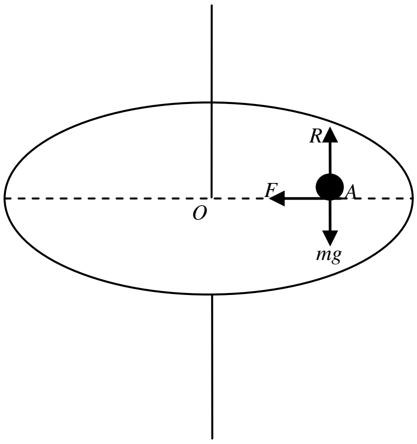
Q	Solution	Mark	Notes
3	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$	M1	used
	$\mathbf{r}_A = (1 + 2t)\mathbf{i} + 5t\mathbf{j} - 4t\mathbf{k}$		
	$\mathbf{r}_B = (3 + t)\mathbf{i} + 3t\mathbf{j} - 5t\mathbf{k}$	A1	either correct, any form
	$\mathbf{r}_B - \mathbf{r}_A = (2 - t)\mathbf{i} - 2t\mathbf{j} - t\mathbf{k}$	M1	
	$AB^2 = x^2 + y^2 + z^2$	M1	
	$AB^2 = (2 - t)^2 + 4t^2 + t^2$	A1	cao
	$(AB^2 = 6t^2 - 4t + 4)$		
	Differentiate	M1	at least 1 power reduced
	$\frac{\mathrm{d}AB^2}{\mathrm{d}t} = 2(2 - t)(-1) + 10t \ \ (= 12t - 4)$		
	$-4 + 2t + 10t = 0$	m1	equating to 0.
	$t = \frac{1}{3}$	A1	cao
	$(\text{least distance})^2 = (2 - \frac{1}{3})^2 + 5(\frac{1}{3})^2$		
	least distance $= \sqrt{\frac{10}{3}} =$ <u>1.83 (m)</u>	A1	cao

Q	Solution	Mark	Notes
4(a)	Conservation of momentum	M1	dimensionally correct
	$12 \times 600 = 1600 \times v$	A1	
	$v = \frac{9}{2}$ (ms^{-1})	A1	allow -ve
4(b)	Energy considerations	M1	
	$E = 0.5\times12\times600^2 + 0.5\times1600\times4.5^2$	A1	both expressions correct, Ft v in (a)
	$E = 2160000 + 16200$		
	$E = \underline{2176200 \text{ (J)}}$	A1	cao
	Energy dissipated by eg sound of cannon firing ignored.	E1	oe
4(c)	Work-energy principle	M1	used
	$F \times d = E$		
	$F \times 1.2 = 16200$		
	$F = \underline{13500 \text{ (N)}}$	A1	cao

Q	Solution	Mark	Notes
5.	Hooke's Law	M1	used
	$30 = \frac{\lambda(0 \cdot 95 - l)}{l}$	A1	
	$70 = \frac{\lambda(1 \cdot 15 - l)}{l}$	A1	
	$\frac{70}{30} = \frac{(1.15 - l)}{(0 \cdot 95 - l)}$	m1	getting to equation with 1 variable
	$7(0.95 - l) = 3(1.15 - l)$		
	$l = \underline{0.8}$	A1	cao
	$\lambda = \underline{160}$	A1	cao

Q	Solution	Mark	Notes
6(a)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$	M1	sin to cos and coefficient multiplied
	$\mathbf{a} = 14\cos 2t\,\mathbf{i} - 18\sin 3t\,\mathbf{j}$	A1	
6(b)	$\mathbf{r} = \int 7\sin 2t\ \mathbf{i} + 6\cos 3t\,\mathbf{j}\ \mathrm{d}t$	M1	sin to cos and coefficient divided.
	$\mathbf{r} = -3.5\cos 2t\,\mathbf{i} + 2\sin 3t\,\mathbf{j} + (\mathbf{c})$	A1	
	$t = 0$, $\mathbf{r} = 0.5\,\mathbf{i} + 3\,\mathbf{j}$	m1	used
	$0.5\,\mathbf{i} + 3\,\mathbf{j} = -3.5\,\mathbf{i} + \mathbf{c}$		
	$\mathbf{c} = 4\,\mathbf{i} + 3\,\mathbf{j}$		
	When $t = \frac{\pi}{2}$	m1	substituted si
	$\mathbf{r} = -3.5\cos\pi\,\mathbf{i} + 2\sin\frac{3}{2}\pi\,\mathbf{j} + 4\,\mathbf{i} + 3\,\mathbf{j}$		
	$\mathbf{r} = (4 + 3.5)\,\mathbf{i} + (3 - 2)\,\mathbf{j}$		
	$\mathbf{r} = \underline{7.5\,\mathbf{i} + \mathbf{j}\text{ (m)}}$	A1	cao
OR			
	$\int_0^{\pi/2} 7\sin 2t\ \mathbf{i} + 6\cos 3t\,\mathbf{j}\ \mathrm{d}t$	(M1)	attempt to integrate
	$= [-3.5\cos 2t\,\mathbf{i} + 2\sin 3t\,\mathbf{j}]^{\pi/2}$	(A1)	correct integration
	$= 3.5\,\mathbf{i} - 2\,\mathbf{j} + 3.5\,\mathbf{i}$	(m1)	correct use of limits $0,\pi/2$
	$\mathbf{r} = 0.5\,\mathbf{i} + 3\,\mathbf{j} + 3.5\,\mathbf{i} - 2\,\mathbf{j} + 3.5\,\mathbf{i}$	(m1)	adding $0.5\,\mathbf{i} + 3\,\mathbf{j}$
	$\mathbf{r} = \underline{7.5\,\mathbf{i} + \mathbf{j}\text{ (m)}}$	(A1)	cao

Q	Solution	Mark	Notes
7.	K. Energy. at $A = 0.5 \times 70 \times v^2$	B1	
	K. Energy. at $A = 35v^2$		
	Let potential energy be 0 at A		
	P. Energy at $B = 70{\times}9.8{\times}(22\text{-}20)$	M1	mgh attempted
	P. Energy at $B = 70{\times}9.8{\times}2$	A1	correct for h=2, 20, 22
	P. Energy at $B = 1372$		
	Minimum K. Energy at $B = 0$		
	WD against resistance = 50×16	B1	
	WD against resistance = 800		
	Work-Energy Principle	M1	at least 3 energies
	$35v^2 = 1372 + 800$	A1	ft one arithmetic slip
	$v = \underline{7.88}$	A1	cao

Q	Solution	Mark	Notes
8			
	Resolve vertically $R = mg$	B1	
	$F = \mu R = 0.72mg$	B1	ft *R*, si
	If particle remains at *A*		
	$F \geq ma$	M1	accept =, used, No extra force
	$0.72mg \geq \frac{mv^2}{1 \cdot 6}$	A1	accept =
	$v^2 \leq 0.72 \times 9.8 \times 1.6$		
	$v \leq \underline{3.36}$	A1	cao, accept =
	Greatest value of v is $\underline{3.36}$		
	$\omega \leq \frac{3 \cdot 36}{1 \cdot 6}$		
	$\omega \leq \underline{2.1 \text{ rads}^{-1}}$	A1B1	accept =, ft *v*
	Greatest value of ω is $\underline{2.1 \text{ rads}^{-1}}$		

Q	Solution	Mark	Notes
9(a)	Conservation of energy	M1	KE and PE
	$0.5\times m\times g + mg\times 4(1 - \cos\theta)$		
	$= 0.5\times m\times v^2$	A1	KE both sides, oe
		A1	correct equation, any form
	$g + 8g(1 - \cos\theta) = v^2$		
	$v^2 = \underline{g(9 - 8\cos\theta)}$	A1	cao, simplified, ISW
9(b)	N2L towards centre of motion	M1	dim correct, 3 terms, *mg*cosθ and *R* opposing
	$mg\cos\theta - R = \frac{mv^2}{4}$	A1	
	$R = mg\cos\theta - \frac{mg}{4}(9 - 8\cos\theta)$	m1	
	$R = \underline{3mg(\cos\theta - 0.75)}$	A1	cao, any form ISW
	P leaves the surface when *R*=0	M1	
	$\cos\theta = \underline{0.75}$	A1	cao
	$v^2 = \underline{g(9 - 8\times 0.75)}$		
	$v^2 = \underline{3g} = \underline{29.4}$	A1	cao