GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C4
0976/01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics C4 June 2017

Solutions and Mark Scheme

1. (*a*) $f(x) \equiv \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x+4)}$ (correct form) M1

$8x^2 + 7x - 25 \equiv A(x+4) + B(x-1)(x+4) + C(x-1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -2$, $C = 3$, $B = 5$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\frac{9x^2 + 5x - 24}{(x-1)^2(x+4)} = \frac{8x^2 + 7x - 25}{(x-1)^2(x+4)} + \frac{x^2 - 2x + 1}{(x-1)^2(x+4)}$ M1

$\frac{x^2 - 2x + 1}{(x-1)^2(x+4)} = \frac{1}{x+4}$ A1

$\frac{9x^2 + 5x - 24}{(x-1)^2(x+4)} = \frac{-2}{(x-1)^2} + \frac{5}{(x-1)} + \frac{4}{(x+4)}$

(f.t. candidate's values for A, B, C) A1

2. (*a*) $6y^5\frac{dy}{dx} - 12x^3 - 9x^2\frac{dy}{dx} - 18xy = 0$

$\left(6y^5\frac{dy}{dx} - 12x^3\right)$ B1

$\left(-9x^2\frac{dy}{dx} - 18xy\right)$ B1

$\frac{dy}{dx} = \frac{6xy + 4x^3}{2y^5 - 3x^2}$

(convincing i.e intermediary line required) B1

(*b*) $y = 0 \Rightarrow x = 2$ or $x = -2$ B1

At $(2, 0)$, $\frac{dy}{dx} = -\frac{8}{3}$ B1

At $(-2, 0)$, $\frac{dy}{dx} = \frac{8}{3}$ B1

3. (*a*) $5\cos^2\theta + 7 \times 2\sin\theta\cos\theta = 3\sin^2\theta$

(correct use of $\sin 2\theta = 2\sin\theta\cos\theta$) M1

An attempt to form a quadratic equation in $\tan\theta$ by dividing throughout by $\cos^2\theta$ and then using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ m1

$3\tan^2\theta - 14\tan\theta - 5 = 0$ (c.a.o.) A1

$\tan\theta = -\dfrac{1}{3}$, $\tan\theta = 5$ (c.a.o.) A1

$\theta = 161{\cdot}57°$ B1

$\theta = 78{\cdot}69°$ B1

Note: F.t. candidate's derived quadratic equation in $\tan\theta$.
Do not award the corresponding B1 if the candidate gives more than one root in that particular branch. Ignore roots outside range.

(*b*) (i) $R = 4$ B1

Correctly expanding $\cos(\phi - \alpha)$ and using either $4\cos\alpha = \sqrt{5}$ **or** $4\sin\alpha = \sqrt{11}$ **or** $\tan\alpha = \dfrac{\sqrt{11}}{\sqrt{5}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 56°$ (c.a.o) A1

(ii) Least value of $\dfrac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6} = \dfrac{1}{4 \times k + 6}$.

($k = \mathbf{1}$ or -1)

(f.t. candidate's value for R) M1

Least value $= \dfrac{1}{10}$ (f.t. candidate's value for R) A1

Corresponding value for $\phi = 56°$ (o.e.)

(f.t. candidate's value for α) A1

4.

Volume $= \pi \int_{\pi/6}^{\pi/3} (\cos x + \sec x)^2 \, dx$ B1

Correct use of $\cos^2 x = \frac{(1 + \cos 2x)}{2}$ M1

Integrand $= \frac{(1 + \cos 2x)}{2} + 2 + \sec^2 x$ (c.a.o.) A1

$\int a \cos 2x \, dx = \frac{a}{2} \sin 2x$ $(a \neq 0)$ B1

$\int b \, dx = bx$ and $\int \sec^2 x \, dx = \tan x$ $(b \neq 0)$ B1

Correct substitution of correct limits in candidate's integrated expression of the form
$px + q\sin 2x + \tan x$ $(p \neq 0, q \neq 0)$ M1

Volume $= \pi \times (4{\cdot}566551037 - 2{\cdot}102853559) = 7{\cdot}74$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. (*a*) $(1 + 4x)^{-1/2} = 1 - 2x + 6x^2 + \ldots$ $(1 - 2x)$ B1
$(6x^2)$ B1

$|x| < {}^1/_4$ or $-{}^1/_4 < x < {}^1/_4$ B1

(*b*) $1 + 4y + 8y^2 = 1 + 4(y + 2y^2)$ M1

$(1 + 4y + 8y^2)^{-1/2} = 1 - 2(y + 2y^2) + 6(y + 2y^2)^2 + \ldots$
(f.t. candidate's expression from part (*a*)) m1

$(1 + 4y + 8y^2)^{-1/2} = 1 - 2y + 2y^2 + \ldots$
(f.t. candidate's expression from part (*a*)) A1

6. (*a*) candidate's x-derivative $= 2at$

candidate's y-derivative $= 3bt^2$ (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{3bt}{2a}$ (o.e.) (c.a.o.) A1

Equation of tangent at P: $y - bp^3 = \frac{3bp}{2a}(x - ap^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$2ay = 3bpx - abp^3$ (convincing) A1

(*b*) Substituting $4a$ for x and $8b$ for y in equation of tangent M1

$16ab = 12abp - abp^3 \Rightarrow p^3 - 12p + 16 = 0$ (convincing) A1

$(p - 2)(p^2 + 2p - 8) = 0$ M1

$(p - 2)(p - 2)(p + 4) = 0$ A1

$p = 2$ corresponds to $(4a, 8b) \Rightarrow p = -4$ (c.a.o.) A1

7. (*a*) $u = \ln x \Rightarrow du = \frac{1}{x}dx$ B1

$dv = x^{-4}dx \Rightarrow v = \frac{1}{-3}x^{-3}$ (o.e.) B1

$\int \frac{\ln x}{x^4}dx = \frac{1}{-3}x^{-3} \times \ln x - \int \frac{1}{-3}x^{-3} \times \frac{1}{x}dx$ (o.e.) M1

$\int \frac{\ln x}{x^4}dx = \frac{1}{-3}x^{-3} \times \ln x - \frac{1}{9}x^{-3} + c$ (c.a.o.) A1

(*b*) $\int x^3(x^2 + 1)^4 dx = \int f(u) \times u^4 \times du$ $(f(u) = pu + q, p \neq 0, q \neq 0)$ M1

$\int x^3(x^2 + 1)^4 dx = \int \frac{(u - 1)}{2} \times u^4 \times du$ A1

$\int (pu^5 + qu^4)\,du = \frac{pu^6}{6} + \frac{qu^5}{5}$ B1

Either: Correctly inserting limits of 1, 2 in candidate's $\frac{pu^6}{6} + \frac{qu^5}{5}$

or: Correctly inserting limits of 0, 1 in candidate's $\frac{p(x^2 + 1)^6}{6} + \frac{q(x^2 + 1)^5}{5}$ m1

$\int_0^1 x^3(x^2 + 1)^4 dx = \frac{43}{20} = 2{\cdot}15$ (c.a.o.) A1

8. (*a*) $\frac{dN}{dt} = k\sqrt{N}$ B1

(*b*) $\int \frac{dN}{\sqrt{N}} = \int k\, dt$ M1

$\frac{N^{1/2}}{1/2} = kt + c$ A1

Substituting 256 for N and 5 for t and 400 for N and 7 for t in candidate's derived equation m1

$32 = 5k + c$, $40 = 7k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c ($k = 4$, $c = 12$) m1

$N = (2t + 6)^2$ (o.e.) (c.a.o.) A1

9. (*a*) **AD** = **AO** + **OD** = – **a** +2**b** B1

Use of **a** + λ**AD** (o.e.) to find vector equation of AD M1

Vector equation of AD: **r** = **a** + λ(– **a** +2**b**)

r = $(1 - \lambda)$**a** + 2λ**b** (convincing) A1

(*b*) **BC** = **BO** + **OC** = 5**a** – **b** B1

Vector equation of BC: **r** = **b** + μ (5**a** – **b**)

r = 5μ**a** +$(1 - \mu)$**b** (o.e.) B1

(*c*) $1 - \lambda = 5\mu$

$2\lambda = 1 - \mu$

(comparing candidate's coefficients of **a** and **b** and an attempt to solve) M1

$\lambda = \frac{4}{9}$ or $\mu = \frac{1}{9}$ (f.t. candidate's derived vector equation of BC) A1

OE = $\frac{5}{9}$**a** + $\frac{8}{9}$**b** (f.t. candidate's derived vector equation of BC) A1

10. $a^2 = 7b^2 \Rightarrow (7k)^2 = 7b^2 \Rightarrow b^2 = 7k^2$ B1

$\therefore$ 7 is a factor of b^2 and hence 7 is a factor of b B1

$\therefore$ a and b have a common factor, which is a contradiction to the original assumption B1