



GCE AS/A Level

0976/01



S17-0976-01

MATHEMATICS – C4
Pure Mathematics

FRIDAY, 16 JUNE 2017 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Express $\frac{8x^2 + 7x - 25}{(x-1)^2(x+4)}$ in terms of partial fractions. [4]

- (b) Use your result to part (a) to express $\frac{9x^2 + 5x - 24}{(x-1)^2(x+4)}$ in terms of partial fractions. [3]

2. The curve C has equation

$$y^6 - 3x^4 - 9x^2y + 48 = 0.$$

- (a) Show that $\frac{dy}{dx} = \frac{6xy + 4x^3}{2y^5 - 3x^2}$. [3]

- (b) Find the gradient of the tangent to C at each of the points where C crosses the x -axis. [3]

3. (a) Show that the equation

$$5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta$$

may be rewritten in the form

$$a\tan^2\theta + b\tan\theta + c = 0,$$

where a, b, c are non-zero constants whose values are to be found.

Hence, find all values of θ in the range $0^\circ \leq \theta \leq 180^\circ$ satisfying the equation

$$5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta. \quad [6]$$

- (b) (i) Express $\sqrt{5}\cos\phi + \sqrt{11}\sin\phi$ in the form $R\cos(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

- (ii) Use your result to part (i) to find the least value of

$$\frac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6}.$$

Write down a value for ϕ for which this least value occurs. [6]

4. The region R is bounded by the curve $y = \cos x + \sec x$, the x -axis and the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$. Find the volume of the solid generated when R is rotated through four right angles about the x -axis. Give your answer correct to two decimal places. [7]

5. (a) Expand $(1 + 4x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2 . State the range of values of x for which your expansion is valid. [3]

- (b) Use your answer to part (a) to expand $(1 + 4y + 8y^2)^{-\frac{1}{2}}$ in ascending powers of y up to and including the term in y^2 . [3]

6. The curve C has the parametric equations $x = at^2$, $y = bt^3$, where a, b are positive constants. The point P lies on C and has parameter p .

- (a) Show that the equation of the tangent to C at the point P is

$$2ay = 3bpx - abp^3. \quad [5]$$

- (b) The tangent to C at the point P intersects C again at the point with coordinates $(4a, 8b)$. Show that p satisfies the equation

$$p^3 - 12p + 16 = 0.$$

Hence find the value of p . [5]

7. (a) Find $\int \frac{\ln x}{x^4} dx$. [4]

- (b) Use the substitution $u = x^2 + 1$ to evaluate

$$\int_0^1 x^3(x^2 + 1)^4 dx. \quad [5]$$

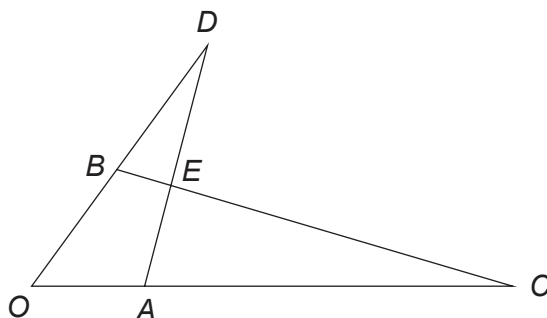
8. The size N of the population of a small island may be modelled as a continuous variable. At time t years, the rate of increase of N is assumed to be directly proportional to the value of $\sqrt{N}$.

- (a) Write down a differential equation satisfied by N . [1]

- (b) When $t = 5$, the size of the population was 256. When $t = 7$, the size of the population was 400. Find an expression for N in terms of t . [6]

TURN OVER

9. In the diagram below, the points O, A, B, C and D are as follows. A lies on OC and $OC = 5OA$. B lies on OD and $OD = 2OB$.



Taking O as origin, the position vectors of A and B are denoted by $\mathbf{a}$ and $\mathbf{b}$ respectively.

- (a) Write down the vector $\mathbf{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Hence show that the vector equation of the line AD may be expressed in the form

$$\mathbf{r} = (1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b}. \quad [3]$$

- (b) Find a similar expression for the vector equation of the line BC . [2]

- (c) The lines AD and BC intersect at the point E . Find the position vector of E in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

10. Complete the following proof by contradiction to show that $\sqrt{7}$ is irrational.

Assume that $\sqrt{7}$ is rational. Then $\sqrt{7}$ may be written in the form $\frac{a}{b}$,

where a, b are integers having no factors in common.

$\therefore a^2 = 7b^2$.

$\therefore a^2$ has a factor 7.

$\therefore a$ has a factor 7 so that $a = 7k$, where k is an integer. [3]

END OF PAPER