GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - M2
0981-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics M2 (June2017)
Markscheme

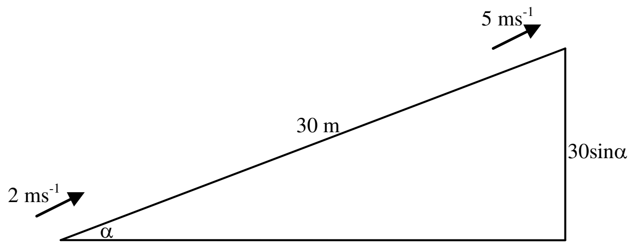
Q	Solution	Mark	Notes
1(a)(i)	$\mathbf{v} = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r}$	M1	differentiation attempted
			Vector required
	$\mathbf{v} = (\sin t + t\cos t)\mathbf{i} + (\cos t - t\sin t)\mathbf{j}$	A1	
	$(\text{mod } \mathbf{v})^2 = (\sin t + t\cos t)^2 + (\cos t - t\sin t)^2$	M1	
	$= \sin^2 t + 2t\sin t\cos t + t^2\cos^2 t$		
	$+ \cos^2 t - 2t\sin t\cos t + t^2\sin^2 t$	A1	Ft similar expressions
	$= 1 + t^2$		
	Speed of $P = \sqrt{1+t^2}$	A1	cao
1(a)(ii)	Momentum vector = $m\mathbf{v}$		
	$= 3[(\sin t + t\cos t)\mathbf{i} + (\cos t - t\sin t)\mathbf{j}]$	B1	ft $\mathbf{v}$(c)
	$= 3(\sin t + t\cos t)\mathbf{i} + 3(\cos t - t\sin t)\mathbf{j}$		
1(b)	At $t = \frac{\pi}{6}$,		
	$\mathbf{r} = \frac{\pi}{6}\sin\frac{\pi}{6}\mathbf{i} + \frac{\pi}{6}\cos\frac{\pi}{6}\mathbf{j}$	B1	
	$\mathbf{r} = \frac{\pi}{12}\mathbf{i} + \frac{\pi\sqrt{3}}{12}\mathbf{j}$		
	If perpendicular, $\mathbf{r}.(b\,\mathbf{i} + \sqrt{3}\,\mathbf{j}) = 0$	M1	
	$(\frac{\pi}{12}\mathbf{i} + \frac{\pi\sqrt{3}}{12}\mathbf{j}).(b\,\mathbf{i} + \sqrt{3}\,\mathbf{j})$		
	$= \frac{\pi}{12}b + \frac{\pi\sqrt{3}}{12}\times\sqrt{3}$	M1A1	method correct, no **i**, **j**
	$\frac{\pi}{12}b + \frac{3\pi}{12} = 0$		
	$b + 3 = 0$		
	$b = \underline{-3}$	A1	cao

Q	Solution	Mark	Notes
2(a)	$x = \int 4t^3 - 6t + 7\,\mathrm{d}t$	M1	at least one power increased.
	$x = t^4 - 3t^2 + 7t + (C)$	A1	correct integration
	When $t = 0$, $x = 5$	m1	initial conditions used
	$C = 5$		
	$x = t^4 - 3t^2 + 7t + 5$		
	When $t = 2$	m1	used
	$x = 2^4 - 3\times2^2 + 7\times2 + 5$		
	$x = 16 - 12 + 14 + 5$		
	$x = \underline{23\text{ (m)}}$	A1	cao
2(b)	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	at least one power decreased.
	$a = 12t^2 - 6$	A1	
	$F = ma = 0.8(12t^2 - 6)$	M1	Ft a
	When $t = 3$		
	$F = 0.8(12\times3^2 - 6)$		
	$F = \underline{81.6\text{ (N)}}$	A1	cao

Q	Solution	Mark	Notes
3(a).	$T = \frac{P}{v}$		
	$T = \frac{12000}{3} = (4000)$	B1	
	N2L	M1	dimensionally correct 4 terms, allow sin/cos
	$T - mg\sin\alpha - R = ma$	A1	
	$4000 - 3000\times9.8\times0.1 - 460 = 3000a$		
	$a = \underline{0.2\ (\text{ms}^{-2})}$	A1	cao
3(b)	N2L	M1	dimensionally correct 4 terms, allow sin/cos
	$a = 0$	M1	
	$T - 10v - mg\sin\alpha - R = 0$		
	$\frac{12000}{v} - 10v - 3000\times9.8\times0.1 - 460 = 0$	A1	
	$\frac{12000}{v} - 10v - 3400 = 0$		
	$12000 - 10v^2 - 3400v = 0$		
	$v^2 + 340v - 1200 = 0$		
	$v = \frac{-340 \pm \sqrt{340^2 + 4\times1200}}{2}$	m1	dep on both M
	$v = \underline{3.49}$	A1	cao answer rounding to 3.5.

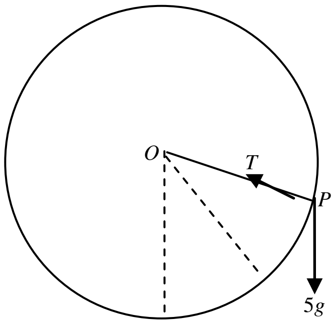
Q	Solution	Mark	Notes
4(a)	initial vertical vel of $P = 15\sin60°$		
	$= \frac{15\sqrt{3}}{2} = 12.99$		
	initial vertical vel of $Q = v\sin30°$	B1	either correct expression
	use of $s = ut + 0.5gt^2$	M1	
	height of P at time $t = \frac{15\sqrt{3}}{2}t - 0.5gt^2$		
	height of Q at time $t = 0.5vt - 0.5gt^2$	A1	either
	For collision		
	$\frac{15\sqrt{3}}{2}t - 0.5gt^2 = 0.5vt - 0.5gt^2$	m1	
	$v = 15\sqrt{3} = 25.98$	A1	accept 26
4(b)	initial horiz vel of $P = 15\cos60°$		
	$= 7.5$		
	initial horiz vel of $Q = 15\sqrt{3}\cos30°$		
	$= 22.5$	B1	either
	For collision,		
	$7.5t + 22.5t = 18$	M1	
	$t = 0.6$ (s)	A1	convincing
4(c)	use of $v=u+at$, $u=\frac{15\sqrt{3}}{2}$ (c), $a=\pm9.8$, $t=0.6$	M1	
	$v = \frac{15\sqrt{3}}{2} - 9.8\times0.6$	A1	Ft u
	$v = 7.1$		
	speed $= \sqrt{7.1^2 + 7.5^2}$	M1	accept candidate's values
	$=$ <u>10.3(ms^{-1})</u>	A1	cao

Q	Solution	Mark	Notes
5			



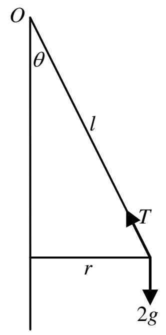
KE at t=0 = 0.5 × 4000 × 2^2	M1A1	v=2 or 5
KE at t=0 = 8000 (J)		
PE at t =0 = 0		
KE at t=8 = 0.5 × 4000 × 5^2		
KE at t=8 = 50000 (J)		
PE at t =8 = 4000 × 9.8 × h	M1	
PE at t =8 = 4000 × 9.8 × 30sinα	A1	
PE at t =8 = 58800 (J)		
WD by engine = 43000 × 8	B1	
WD by engine = 344000 (J)		
Work-energy principle	M1	KE, PE and WD(2 terms)
8000 + 344000 = WD + 50000 + 58800	A1	correct equation
WD = 243200 (J)	A1	cao

Q	Solution	Mark	Notes
6			



Q	Solution	Mark	Notes
6(a)	conservation of energy	M1	KE and PE in equation
	$0.5mu^2 - mgl\cos60° = 0.5mv^2 - mgl\cos\theta$	A1A1	
	$v^2 = u^2 - 0.8g + 1.6g\cos\theta$	A1	cao
	$v^2 = u^2 - 7.84 + 15.68\cos\theta$		
6(b)	N2L towards centre	M1	dim correct equation T and $5g\cos\theta$ opposing
	$T - 5g\cos\theta = \dfrac{5v^2}{0.8}$	A1	
	$T = 5g\cos\theta + \dfrac{5}{0.8}(u^2 - 0.8g + 1.6g\cos\theta)$	m1	subt v^2 equivalent expressions
	$T = 6.25u^2 - 5g + 15g\cos\theta$	A1	cao, any correct expression
	$T = 6.25u^2 - 49 + 147\cos\theta$		
6(c)	For complete circles,		
	$T \geq 0$ when $\theta = 180°$, ($\cos\theta = -1$).	M1	
	$6.25u^2 \geq 49 + 147$		
	$u^2 \geq 31.36$		
	$u \geq 5.6$	A1	cao
6(d)	For complete circles,		
	$v^2 \geq 0$ when $\theta = 180°$, ($\cos\theta = -1$).	M1	
	$u^2 \geq 7.84 + 15.68$		
	$u^2 \geq 23.52$		
	$u \geq 4.85$	A1	cao

Q	Solution	Mark	Notes
7.			



7(a)	Resolve vertically	M1	
	$T\cos\theta = 2g$	A1	allow m
	N2L towards centre of motion	M1	
	$T\sin\theta = 2r\omega^2$	A1	
	$T\sin\theta = 2l\sin\theta\ \omega^2$	A1	use of $r=l\sin\theta$
	$T = 2l\omega^2$		
	$2l\ \omega^2\cos\theta = 2g$		
	$\cos\theta = \dfrac{g}{l\omega^2}$	A1	convincing
7(b)(i)	$T\cos\theta = 2g,\ T = 20g$		
	$\cos\theta = \underline{0.1}$	B1	
7(b)(ii)	$\cos\theta = 0.1$ and $\omega^2 = 3g$, $\cos\theta = \dfrac{g}{l\omega^2}$		
	$0.1 = \dfrac{g}{l \times 3g}$	M1	or $20g=2l\text{x}3g$
	$l = \dfrac{10}{3}$	A1	convincing
7(b)(iii)	Hooke's Law		
	$T = \dfrac{\lambda x}{natural\ length}$	M1	used, condone natural length=10/3, but x not10/3or 3
	$20g = \dfrac{\lambda(\frac{10}{3} - 3)}{3}$	A1	one of 10/3-3 or 3 correct
	$\lambda = \underline{180g} = \underline{1764}$	A1	cao

7(b)(iv) EE $= \dfrac{\lambda x^2}{2(nat\ len)}$	M1	used
EE $= \dfrac{1764}{2 \times 3 \times 3^2}$		
EE $= \dfrac{98}{3}$ = <u>32.67 (J)</u>	A1	cao