



GCE AS/A Level

0981/01



MATHEMATICS – M2
Mechanics

TUESDAY, 20 JUNE 2017 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9.8 ms^{-2} .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The position vector of a particle P at time t seconds is given by

$$\mathbf{r} = t \sin t \mathbf{i} + t \cos t \mathbf{j}.$$

- (a) (i) Find the velocity vector of P and an expression for the speed of P at time t seconds in its simplest form.
- (ii) Given that the mass of P is 3 kg, write down the momentum vector of P at time t seconds. [6]
- (b) At time $t = \frac{\pi}{6}$, the vector $b\mathbf{i} + \sqrt{3}\mathbf{j}$ is perpendicular to \mathbf{r} . Find the value of b . [5]

2. A particle P , of mass 0.8 kg, moves along the x -axis so that its velocity at time t seconds is $v \text{ ms}^{-1}$, where $v = 4t^3 - 6t + 7$. Given that the displacement of P is 5 m from the origin when $t = 0$, find

- (a) the displacement of P from the origin when $t = 2$, [5]
- (b) the force acting on P when $t = 3$. [4]

3. A vehicle of mass 3000 kg has an engine that is capable of producing power up to 12 000 W. The vehicle moves up a slope inclined at an angle α to the horizontal, where $\sin \alpha = 0.1$. The resistance to motion experienced by the vehicle is constant at 460 N.

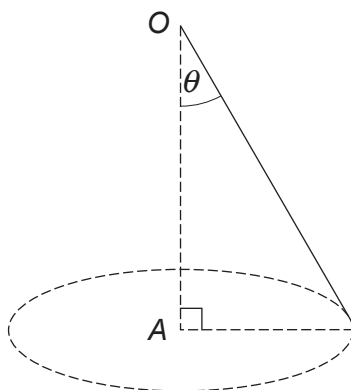
- (a) Find the maximum acceleration of the vehicle when its velocity is 3 ms^{-1} . [4]
- (b) The vehicle now travels at a velocity of $v \text{ ms}^{-1}$ against an additional braking force of $10v \text{ N}$. The other resistance to motion remains constant at 460 N. Determine the maximum value of v . Give your answer correct to 2 decimal places. [5]

4. A and B are points a distance 18 m apart on horizontal ground. An object P is projected from A towards B with velocity 15 ms^{-1} at an angle of 60° to the horizontal. Simultaneously, another object Q is projected from B towards A with velocity $v \text{ ms}^{-1}$ at an angle of 30° to the horizontal. The objects collide.

- (a) Find the value of v . [5]
- (b) Show that the time from projection to collision is 0.6 seconds. [3]
- (c) Determine the speed of the object P just before collision. [4]

5. A vehicle of mass 4000 kg is moving up a hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$. At time $t = 0 \text{ s}$, the speed of the vehicle is 2 ms^{-1} . At time $t = 8 \text{ s}$, the vehicle has travelled 30 m up the hill from its initial position and its speed is 5 ms^{-1} . The vehicle's engine is working at a constant rate of 43 000 W. Find the total work done against the resistive forces during this 8 second period. [8]

6. A particle P , of mass 5 kg , is attached to one end of a light inextensible string of length 0.8 m . The other end of the string is attached to a fixed point O . Initially, the particle P is held at rest with the string OP taut and inclined at an angle of 60° to the downward vertical through O . The particle P is then projected with speed $u\text{ ms}^{-1}$ in a downward direction perpendicular to the string, so that P starts to describe a vertical circle with centre O . When the string OP is inclined at an angle θ to the downward vertical, the speed of P is $v\text{ ms}^{-1}$.
- Find, in terms of u and θ , an expression for v^2 . [4]
 - Find, in terms of u and θ , an expression for the tension in the string when OP makes an angle θ with the downward vertical. [4]
 - Determine the least value of u so that the particle describes complete circles. [2]
 - Suppose that the string is replaced by a light rod. Determine the least value of u so that the particle describes complete circles. [2]
7. A particle of mass 2 kg is suspended from a fixed point O by means of an elastic string of natural length 3 m and modulus of elasticity $\lambda\text{ N}$. The particle describes a horizontal circle with constant angular speed $\omega\text{ rad s}^{-1}$, with the string being of constant length $l\text{ m}$, where $l > 3$. The centre of the circle A is vertically below O and the angle between the string and the downward vertical is θ .



- Show that $\cos\theta = \frac{g}{l\omega^2}$. [6]
- Given that the tension in the string is $20g\text{ N}$ and $\omega^2 = 3g$,
 - find the value of $\cos\theta$,
 - show that $l = \frac{10}{3}$,
 - calculate the value of λ ,
 - find the elastic energy in the string. [8]

END OF PAPER