GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - S2
0984-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Ques	Solution	Mark	Notes
1(a)	$E(X) = 2.0$, $E(Y) = 1.6$	**B1**	si
	$E(W) = E(X)E(Y)$	**M1**	
	$= 3.2$	**A1**	
	$Var(X) = 1.2$, $Var(Y) = 1.28$	**B1**	si
	$E(X^2) = Var(X) + [E(X)]^2 = 5.2$	**M1A1**	
	$E(Y^2) = Var(Y) + [E(Y)]^2 = 3.84$	**A1**	
	$Var(W) = E(X^2)E(Y^2) - [E(X)E(Y)]^2$	**M1**	Allow
	$= 9.73$	**A1**	
(b)	$P(W = 0) = P\{(X = 0) \cup (Y = 0)\}$	**M1**	$P(W = 0) = 1 - P(X \geq 0)P(Y \geq 0)$
	$= P(X = 0) + P(Y = 0) - P\{(X = 0) \cap (Y = 0)\}$	**m1**	$= 1 - (1 - P(X = 0))(1 - P(Y = 0))$
	$= 0.6^5 + 0.8^8 - 0.6^5 \times 0.8^8$	**A1**	$= 1 - (1 - 0.6^5)(1 - 0.8^8)$
	$= 0.232$	**A1**	$= 0.232$
2	Under H_0, the number, X, of breakdowns in 100 days is Poi(80) which is approx N(80,80)	**B1B1**	Award M1A0 for an incorrect or no continuity correction and FT for the following marks
	$z = \dfrac{64.5 - 80}{\sqrt{80}}$	**M1A1**	$64 \to z = -1.79 \to p\text{-value} = 0.0367$
	$= -1.73$	**A1**	$63.5 \to z = -1.84 \to p\text{-value} = 0.0329$
	p-value $= 0.0418$	**A1**	
	There is strong evidence to conclude that the mean number of breakdowns per day has been reduced.	**A1**	FT the p-value
3(a)	90th percentile $= \mu + 1.282\sigma$	**M1**	
	$= 128$	**A1**	
(b)	Let $X =$ weight of an apple, $Y =$ weight of a pear		
	Let S denote the sum of the weights of 10 apples		
	Then $E(S) = 1100$	**B1**	
	$Var(S) = 10 \times 14^2$	**M1**	
	$= 1960$	**A1**	
	$z = \dfrac{1000 - 1100}{\sqrt{1960}}$	**m1**	
	$= (-)2.26$	**A1**	
	Prob $= 0.01191$	**A1**	
(c)	Let $U = X_1 + X_2 + X_3 - Y_1 - Y_2$	**M1**	si, condone incorrect notation
	$E(U) = 3 \times 110 - 2 \times 160 = 10$	**A1**	
	$Var(U) = 3 \times 14^2 + 2 \times 16^2 = 1100$	**M1A1**	
	We require $P(U > 0)$		
	$z = \dfrac{0 - 10}{\sqrt{1100}}$	**m1**	
	$= (-)0.30$	**A1**	
	Prob $= 0.6179$	**A1**	

Ques	Solution	Mark	Notes
4(a)	Let x,y denote distance travelled by models A,B respectively. $$\bar{x} = 166.9; \; \bar{y} = 163.9$$	**B1 B1**	
	Standard error $= \sqrt{\dfrac{2 \times 2.5^2}{8}}$ (=1.25)	**M1A1**	
	95% confidence limits are $$166.9 - 163.9 \pm 1.96 \times 1.25$$ giving [0.55,5.45]	**M1A1** **A1**	
(b)	The lower end of the interval will be 0 if $$1.25z = 3$$ $$z = 2.4$$ Tabular value = 0.008(2) cao Smallest confidence level = 98.4%	**M1** **A1** **A1** **A1**	FT their SE and $\bar{x}, \bar{y}$ (for the first two marks only)
5(a)(i)	Under H_0, X is B(50,0.75) Since $p > 0.5$, we consider X' which is B(50,0.25) $\quad P(X \le 31) = P(X' \ge 19) = 0.0287$ $\quad P(X \ge 44) = P(X' \le 6) = 0.0194$ $\quad$ Significance level = 0.0481	**B1** **M1** **A1** **A1** **A1**	si
(ii)	If $p = 0.5$, P(Accept H_0) = P(32 $\le X \le$ 43) $\qquad\qquad = 1 - 0.9675 = 0.0325$	**M1** **A1**	
(b)(i)	Let Y now denote the number of heads so that under H_0 , Y is B(200,0.75) $\cong$ N(150,37.5) $$z = \frac{139.5 - 150}{\sqrt{37.5}}$$ $\qquad = (-)1.71$ Tabular value = 0.0436 $\quad$ p-value = 0.0872 (accept 0.0873)	**B1** **M1A1** **A1** **A1** **A1**	Award M1A0 for incorrect or no continuity correction but FT for following marks $139 \to z = -1.80 \to p$-value = 0.0359 $138.5 \to z = -1.88 \to p$-value = 0.0301 Penultimate A1 for doubling line above
(ii)	There is insufficient evidence to reject H_0.	**A1**	FT the p-value

Ques	Solution	Mark	Notes
6(a)(i)	$f(x) = \dfrac{1}{b-a} \, , \; a \le x \le b$ $= 0$ otherwise	**B1**	Allow <
(ii)	$E(X^2) = \dfrac{1}{b-a} \int x^2 \mathrm{d}x$	**M1**	
	$= \dfrac{1}{b-a} \left[\dfrac{x^3}{3} \right]_a^b$	**A1**	Condone omission of limits
	$= \dfrac{b^3 - a^3}{3(b-a)}$	**A1**	
	$= \dfrac{(b-a)(b^2 + ab + a^2)}{3(b-a)}$	**A1**	
	$= \dfrac{(b^2 + ab + a^2)}{3}$		
(iii)	$\mathrm{Var}(X) = E(X^2) - \left(E(X) \right)^2$	**M1**	
	$= \dfrac{b^2 + ab + a^2}{3} - \left(\dfrac{a^2 + 2ab + b^2}{4} \right)$	**A1**	
	$= \dfrac{4b^2 + 4ab + 4a^2 - 3a^2 - 6ab - 3b^2}{12}$	**A1**	Convincing
	$= \dfrac{(b-a)^2}{12}$		
(b)(i)	$E(Y) = \dfrac{1}{b-a} \int \dfrac{1}{x} \mathrm{d}x$	**M1**	
	$= \dfrac{1}{b-a} \left[\ln x \right]_a^b$	**A1**	Condone omission of limits
	$= \dfrac{\ln b - \ln a}{b-a}$	**A1**	
(ii)	$P(Y \le y) = P\left(\dfrac{1}{X} \le y \right)$	**M1**	
	$= P\left(X \ge \dfrac{1}{y} \right)$	**A1**	
	$= \dfrac{b - \dfrac{1}{y}}{b - a}$		

Ques	Solution	Mark	Notes
(iii)	PDF = derivative of above line $$= \frac{1}{(b-a)y^2}$$	**M1** **A1**	