GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – C3 (LEGACY)
0975-01

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics C3 May 2018

Solutions and Mark Scheme

1. (*a*)

0	1		
0·25	1·015747709		
0·5	1·133148453		
0·75	1·524817911		
1	2·718281828	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·25$ M1

$I \approx \frac{0·25}{3} \times \{1 + 2·718281828 + 4(1·015747709 + 1·524817911) + 2(1·133148453)\}$

$I \approx 16·14684121 \times 0·25 \div 3$

$I \approx 1·345570101$

$I \approx 1·34557$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^1 e^{x^3 - 1}\,dx = \frac{1}{e} \times \int_0^1 e^{x^3}\,dx$ (o.e.) M1

$\int_0^1 e^{x^3 - 1}\,dx = 0·495$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $3(1 + \cot^2\theta) + 6\cot\theta = 8 - 5\cot^2\theta$

(correct use of $\text{cosec}^2\theta = 1 + \cot^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = candidate's coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant m1

$8\cot^2\theta + 6\cot\theta - 5 = 0 \Rightarrow (2\cot\theta - 1)(4\cot\theta + 5) = 0$

$\Rightarrow \cot\theta = \frac{1}{2}, \cot\theta = -\frac{5}{4}$

$\Rightarrow \tan\theta = 2, \tan\theta = -\frac{4}{5}$ (c.a.o.) A1

$\theta = 63{\cdot}43°, 243{\cdot}43°$ B1

$\theta = 141{\cdot}34°, 321{\cdot}34°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta = +, -$, f.t. for 3 marks, $\tan\theta = -, -$, f.t. for 2 marks
$\tan\theta = +, +$, f.t. for 1 mark

(*b*) $\sec\phi \geq 1, \tan\phi \geq 0$ and thus $\sec\phi + 2\tan\phi$ cannot be less than 1 E1

3. $\frac{d}{dx}(x^5) = 5x^4$, $\frac{d}{dx}(17) = 0$ B1

$\frac{d}{dx}(4xy^2) = (4x)(2y)\frac{dy}{dx} + 4y^2$ B1

$\frac{d}{dx}(-2y^3) = -6y^2\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{4y^2 + 5x^4}{6y^2 - 8xy}$ (o.e.) (c.a.o.) B1

4. (*a*) candidate's x-derivative = $\frac{1}{t}$ B1

candidate's y-derivative = $16t^3 - 6t$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = 16t^4 - 6t^2$ (c.a.o.) A1

(*b*) (i) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 64t^3 - 12t$ B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = 64t^4 - 12t^2$ (c.a.o.) A1

(ii) An attempt to solve $64t^4 - 12t^2 - 1 = 0$ as a quadratic in t^2 M1

$(4t^2 - 1)(16t^2 + 1) = 0 \Rightarrow t^2 = {}^1/_4$ $(t^2 = -{}^1/_{16})$ A1

$t = {}^1/_2$ (c.a.o.) A1

5. (*a*) $\frac{d[(2x-5)e^{2x}]}{dx} = e^{2x} \times f(x) + (2x-5) \times g(x)$

$(f(x) \neq 1,\ g(x) \neq 1)$ M1

$\frac{d[(2x-5)e^{2x}]}{dx} = e^{2x} \times 2 + (2x-5) \times 2e^{2x}$ A1

$e^{2x} \times 2 + (2x-5) \times 2e^{2x} + 12 = 0$ (o.e.)

(f.t. one slip in coefficients) A1

$e^{2x} \times (x-2) + 3 = 0$ (convincing) A1

(*b*) $x_0 = 2$

$x_1 = 1{\cdot}945053083$ (x_1 correct, at least 4 places after the point) B1

$x_2 = 1{\cdot}938670473$

$x_3 = 1{\cdot}93788257$

$x_4 = 1{\cdot}937784608 = 1{\cdot}9378$ (x_4 correct to 4 decimal places) B1

Let $g(x) = e^{2x} \times (x-2) + 3$

An attempt to check values or signs of $g(x)$ at $x = 1{\cdot}93775$

$x = 1{\cdot}93785$ M1

$g(1{\cdot}93775) = -8{\cdot}73 \times 10^{-4}$, $g(1{\cdot}93785) = 3{\cdot}35 \times 10^{-3}$ A1

Change of sign $\Rightarrow \alpha = 1{\cdot}9378$ (correct to four decimal places) A1

6. (*a*) (i) $\frac{dy}{dx} = \frac{f(x)}{8 + 7x - 4x^3}$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = \frac{7 - 12x^2}{8 + 7x - 4x^3}$ A1

(ii) $\frac{dy}{dx} = \frac{1}{3} \times (5 - 9x^2)^{-2/3} \times g(x)$ $(g(x) \neq 1)$ M1

$\frac{dy}{dx} = (5 - 9x^2)^{-2/3} \times (-6x)$ A1

(iii) $\frac{dy}{dx} = \frac{(4 - 3\cos x) \times m\cos x - (2 + 5\sin x) \times k\sin x}{(4 - 3\cos x)^2}$

($m = \mathbf{5}, -5$ $k = \mathbf{3}, -3$) M1

$\frac{dy}{dx} = \frac{(4 - 3\cos x) \times 5\cos x - (2 + 5\sin x) \times 3\sin x}{(4 - 3\cos x)^2}$ A1

$\frac{dy}{dx} = \frac{20\cos x - 6\sin x - 15}{(4 - 3\cos x)^2}$ (c.a.o.) A1

(*b*) $5x = \tan y \Rightarrow 5 = \sec^2 y \times \frac{dy}{dx}$ B1

$5 = (1 + \tan^2 y) \times \frac{dy}{dx}$ B1

$5 = [1 + (5x)^2] \times \frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{5}{1 + 25x^2}$ B1

7. (*a*) (i) $\displaystyle\int \frac{5}{e^{3-4x}}\,dx = \int 5\,e^{4x-3}\,dx = k \times 5 \times e^{4x-3} + c$

($k = 1, 4, -{}^1/_4, {}^1/_4$) M1

$\displaystyle\int \frac{5}{e^{3-4x}}\,dx = {}^5/_4 \times e^{4x-3} + c$ (o.e.) A1

(ii) $\displaystyle\int \frac{6}{9x-4}\,dx = k \times 6 \times \ln|9x-4| + c$ ($k = 1, {}^1/_9, 9$) M1

$\displaystyle\int \frac{6}{9x-4}\,dx = \frac{2}{3} \times \ln|9x-4| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) (i) $\displaystyle\int \cos\left(2x - \frac{\pi}{6}\right)dx = k \times \sin\left(2x - \frac{\pi}{6}\right)$

($k = 1, 2, {}^1/_2, -{}^1/_2$) M1

$\displaystyle\int \cos\left(2x - \frac{\pi}{6}\right)dx = {}^1/_2 \times \sin\left(2x - \frac{\pi}{6}\right)$ A1

A correct method for substitution of the correct limits in an expression of the form $m \times \sin\left(2x - \frac{\pi}{6}\right)$ M1

$$\int_{\pi/3}^{\pi/2} \cos\left(2x - \frac{\pi}{6}\right)dx = -\frac{1}{4}$$

(f.t. only for solutions of $-\frac{1}{2}, -1$ from $k = 1$, $k = 2$, respectively) A1

Note: Answer only with no working shown earns 0 marks

(ii) $\cos(2x - \pi/6) < 0$ when $x \in (\pi/3, \pi/2)$ (o.e.)
and thus the integral will have a negative value E1

8. (*a*) Choice of θ, ϕ in **different quadrants** such that $\sin\theta = \sin\phi$ M1
$\sin 2\theta \neq \sin 2\phi$ (including correct evaluations) A1

(*b*) (i) $|x - 16| = 5|x - 7|$ B1
(ii) Trying to solve $x - 16 = 5(x - 7)$ M1
Trying to solve $x - 16 = -5(x - 7)$ M1
(f.t. candidate's values for a, b ($b > 0$), c for both M marks)
$x = 4{\cdot}75$, $x = 8{\cdot}5$ (c.a.o.) A1
F is either 11·25 km or 7·5 km from T
(f.t candidate's derived positive values for x if the first three marks have been awarded) A1

Alternative mark scheme for first three marks of (ii)
$(x - 16)^2 = 5^2 \times (x - 7)^2$ (attempting to square both sides) M1
$24x^2 - 318x + 969 = 0$
(o.e., at least 2 coefficients correct) M1
(f.t. candidate's values for a, b ($b > 0$), c for both M marks)
$x = 4{\cdot}75$, $x = 8{\cdot}5$ (c.a.o.) A1

9. (*a*) $y = 4 - \dfrac{7}{2 - 3x} \Rightarrow 4 \pm y = \pm \dfrac{7}{2 - 3x}$ (separating variables) M1

$2 - 3x = \pm \dfrac{7}{(4 \pm y)}$ m1

$x = \dfrac{1}{3}\left(2 - \dfrac{7}{(4 - y)}\right)$ (c.a.o.) A1

$f^{-1}(x) = \dfrac{1}{3}\left(2 - \dfrac{7}{(4 - x)}\right)$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = [0{\cdot}5, 4)$
[0·5 B1
4) B1

10. (*a*) $D(fg) = (0, \infty)$ B1

(*b*) (i) $fg(x) = (5 - 3x)^2 + 2(5 - 3x) - 24$ B1
(ii) Putting expression for $fg(x)$ equal to 200, collecting terms and setting up a quadratic in x of the form $mx^2 + nx + p = 0$ M1
An attempt to solve quadratic equation in x either by using the quadratic formula or by getting the quadratic expression into the form $(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and $b \times d$ = candidate's constant m1
$9x^2 - 36x - 189 = 0 \Rightarrow x^2 - 4x - 21 = 0 \Rightarrow$
$(x + 3)(x - 7) = 0$ (o.e.) $\Rightarrow x = -3, 7$ (c.a.o.) A1
Reject $x = -3$, thus $x = 7$ (f.t. only for $x = -7, 3$) A1