GCE A LEVEL MARKING SCHEME

SUMMER 2018

A LEVEL (NEW)
MATHEMATICS – UNIT 3 PURE MATHEMATICS B
1300U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE Mathematics – A2 Unit 3 Pure Mathematics B

SUMMER 2018 MARK SCHEME

Q	Solution	Mark	Notes
1	Either $2x + 1 = 3(x - 2)$	M1	Attempt to equate both sides +ve
	$2x + 1 = 3x - 6$		
	$x = 7$	A1	
	OR $2x + 1 = -3(x - 2)$	m1	
	$2x + 1 = -3x + 6$		
	$x = 1$	A1	

Or

	Solution	Mark	Notes
	$(2x + 1)^2 = 9(x - 2)^2$	(M1)	
	$5x^2 - 40x + 35 = 0$	(A1)	any correct equation
	$x^2 - 8x + 7 = 0$		
	$(x - 7)(x - 1) = 0$	(m1)	oe
	$x = 1, 7$	(A1)	both solutions

If considering:

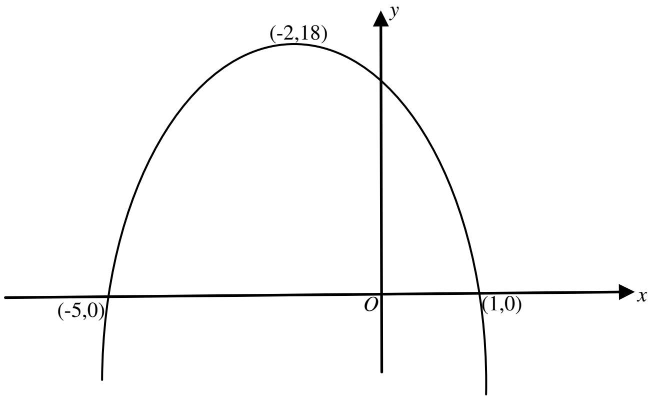
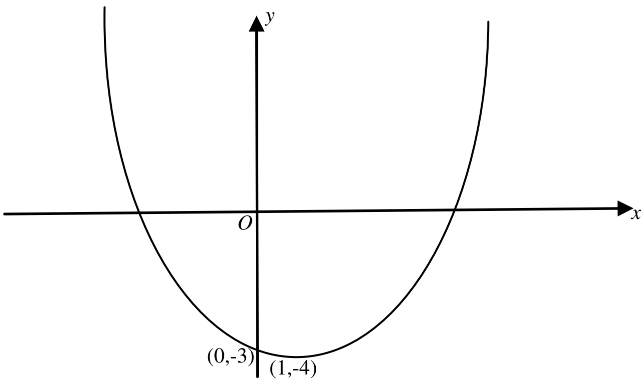
$x < -1/2$ (both sides negative),

$-1/2 \le x < 2$ (LHS negative, RHS positive),

$x \ge 2$ (both sides positive),

give M1, then m1 if all values considered, A1 for 1 and A1 for 7, extra solution/s -1 however many.

Q	Solution	Mark	Notes
2(a)	$s = r\theta$	M1	used
	$5 = 4\theta$		
	$\theta = 1.25^c$	A1	condone 71.62^o, 5.033
2(b)	Area of sector $OAB = \frac{1}{2} \times r^2\theta$	M1	used
	Area of sector $OAB = \frac{1}{2} \times 4^2 \times 1.25$		
	Area of sector $OAB = 10$ (cm^2)	A1	ft θ, accept 40.27

Q	Solution	Mark	Notes
3(a)		B1	correct shape (hill) axes required
		B1	(-2, 18) as max
		B1	(-5, 0), (1, 0)
3(b)		B1	correct shape(cup) axes required
		B1	(1, -4) as min
		B1	(0, -3)

Q	Solution	Mark	Notes
4	$2\tan^2\theta + 2\tan\theta - (1 + \tan^2\theta) = 2$	M1	oe si
	$\tan^2\theta + 2\tan\theta - 3 = 0$		
	$(\tan\theta - 1)(\tan\theta + 3) = 0$	m1	$(\tan\theta + 1)(\tan\theta - 3)$
	$\tan\theta = 1, -3$	A1	cao
	$\theta = 45°, 225°$	B1	ft $\tan\theta$
	$\theta = 108.43°, 288.43°$	B1	

Ignore all roots outside range. For each branch, award B0 if extra root/s present.

2+ve roots ft for B1

2-ve roots ft for B1

Q	Solution	Mark	Notes
5(a)	$\frac{3x}{(x-1)(x-4)^2} = \frac{A}{(x-1)} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2}$		
	$3x = A(x-4)^2 + B(x-1)(x-4) + C(x-1)$	M1	RHS over common denominator
	$x = 4$, $12 = 3C$, $C = 4$	m1	compare coefficients or substitute values.
	$x = 1$, $3 = 9A$, $A = \frac{1}{3}$		
	coefficient x^2, $0 = A + B$, $B = -\frac{1}{3}$	A1	all 3 values correct
5(b)	$I = \int_5^7 \frac{1}{3(x-1)} - \frac{1}{3(x-4)} + \frac{4}{(x-4)^2} dx$	M1	attempt to integrate partial Fractions
	$I = \left[\frac{1}{3}\ln\lvert x-1\rvert - \frac{1}{3}\ln\lvert x-4\rvert - \frac{4}{(x-4)}\right]_5^7$	A1	ft any one term correct
		A1	ft all correct integration
	$I = (\frac{1}{3}\ln 6 - \frac{1}{3}\ln 3 - \frac{4}{3}) - (\frac{1}{3}\ln 4 - 4)$	m1	correct use of correct limits
	$I = \frac{1}{3}(8 - \ln 2) = 2.436$ (3d.p. required)	A1	cao

Q	Solution	Mark	Notes
6	$(1-4x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-4) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-4x)^2$	B1	2 correct unsimplified terms
	$= 1 + 2x + 6x^2 + \ldots$	B1	all simplified terms correct
	Expansion is valid when $\lvert 4x\rvert < 1$		
	Expansion is valid when $\lvert x\rvert < \frac{1}{4}$	B1	oe
	When $x = \frac{1}{13}$,		
	$\frac{1}{\sqrt{1-\frac{4}{13}}} \cong 1 + 2\times\frac{1}{13} + 6\times\left(\frac{1}{13}\right)^2$	M1	attempt to substitute both sides.
	$\frac{\sqrt{13}}{3} \cong \frac{201}{169}$		
	$\sqrt{13} \cong \frac{603}{169}$ or $\frac{2197}{603}$	A1	

Q	Solution	Mark	Notes
7	$\sin x \cong x$, $\cos x \cong 1 - \frac{1}{2}x^2$	M1	used
	$x + 1 - \frac{1}{2}x^2 = \frac{1}{2}$		
	$x^2 - 2x - 1 = 0$	A1	oe
	$x = \frac{2 \pm \sqrt{2^2 + 4}}{2}$		
	$x = 1 - \sqrt{2}$ (= -0.4142)	A1	cao

Q	Solution	Mark	Notes
8	$a + 6d = 71$	B1	
	$\frac{7}{2}(2a + 6d) = 329$	B1	
	$a + 3d = 47$		
	$a + 6d = 71$		
	$3d = 24$		
	$d = 8$	B1	
	$a = 23$	B1	
	The numbers are 23, 31, 39, 47, 55, 63, 71.	B1	

Q	Solution	Mark	Notes
9(a)	The sum to n terms of a series is $S_n = \frac{a(1-r^n)}{(1-r)}$		
	The sum to infinity is $\lim_{n\to\infty} S_n$.		
	This only converges if $\lim_{n\to\infty} r^n$ converges.		
	Hence the sum to infinity of a GP		
	only converges if $\lvert r \rvert < 1$	B1	oe eg. terms increasing
9(b)	For W, the k^{th} term T_k is $(2r^{k-1})^2 = 4r^{2k-2}$.		
	The $(k+1)^{th}$ term is $(2r^k)^2 = 4r^{2k}$.		
	$\frac{T_{k+1}}{T_k} = \frac{4r^{2k}}{4r^{2k-2}} = r^2$ for all values of k.		
	Therefore W is a GP.	B1	common ratio r^2
	For V, 1^{st} term is 2, common ratio r		
	For W, 1^{st} term is 4, common ratio r^2	B1	si
	$S_V = \frac{2}{1-r}$, $S_W = \frac{4}{(1-r^2)}$	B1	either correct
	$S_W = 3S_V$	M1	used
	$\frac{4}{(1-r^2)} = 3(\frac{2}{1-r})$		
	$\frac{4}{(1+r)(1-r)} = 3(\frac{2}{1-r})$		
	$\frac{2}{(1+r)} = 3\ (r \neq 1)$	A1	oe. ft W eg quadratic equation
	$2 = 3 + 3r$		
	$r = -\frac{1}{3}$	A1	cao

Q	Solution	Mark	Notes
9(c)	Total savings T = $5000[(1.03)+(1.03)^2$ $+(1.03)^3+\ldots..+(1.03)^{20}]$	M1	si
	T = $\dfrac{5000(1\cdot03)(1-1\cdot03^{20})}{1-1\cdot03}$	m1	
	T = 138382 (£)	A1	

Q	Solution	Mark	Notes
10(a)	$x = 2\cos^2\theta - 1$	M1	$\cos 2\theta = 2\cos^2\theta - 1$
	$x = 2y^2 - 1$	A1	isw
	$2y^2 = x + 1$		
10(b)	$\cos 2\theta - \cos\theta + 1 = 0$	M1	
	$2\cos^2\theta - 1 - \cos\theta + 1 = 0$	m1	
	$\cos\theta(2\cos\theta - 1) = 0$	A1	si
	$\cos\theta = \frac{1}{2}, 0$	A1	
	$\theta = \frac{\pi}{3}, \frac{\pi}{2}$		answer given
	Co-ordinates are $P(-\frac{1}{2}, \frac{1}{2})$ and $Q(-1, 0)$	B1	
	<u>Alternative solution</u>		
	Using $x - y + 1 = 0$ and $2y^2 = x + 1$	(M1)	attempt to solve simultaneously
	$2(x + 1)^2 = x + 1$, $2x^2 + 3x + 1 = 0$	(m1)	eliminate one variable
	$(2x + 1)(x + 1) = 0$, $x = -\frac{1}{2}, -1$		
	$y = \frac{1}{2}, y = 0,$	(A1)	
	$\cos\theta = \frac{1}{2}, \cos\theta = 0$	(A1)	si both
	$\theta = \frac{\pi}{3}, \theta = \frac{\pi}{2},$		answer given
	$P(-\frac{1}{2}, \frac{1}{2})$, $Q(-1, 0)$	(B1)	

Accept $P(-\frac{1}{2}, \frac{1}{2})$, $Q(-1, 0)$ (B1), verification for P M1 A1, verification for Q m1 A1.

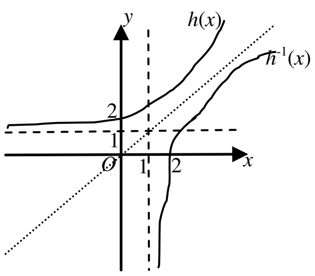
Q	Solution	Mark	Notes
10(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} / \frac{\mathrm{d}x}{\mathrm{d}\theta}$	M1	used
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -2\sin 2\theta$	A1	
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -\sin\theta$	A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\sin\theta}{-2\sin 2\theta} = (\frac{1}{4\cos\theta})$		
	Grad of tgt at $P = \frac{1}{2}$	A1	
	Equ of tgt at P is $y - \frac{1}{2} = \frac{1}{2}(x + \frac{1}{2})$	A1	
	Equ of tgt at P is $4y = 2x + 3$		
	Grad of tgt at Q is undefined.		
	Equ of tgt at Q is $x = -1$	A1	
	Point of intersection is $(-1, \frac{1}{4})$	A1	

OR (first 3 marks)

	$2\times 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 1$	(M1)	attempt implicit differentiation
		(A1)	$2y\frac{\mathrm{d}y}{\mathrm{d}x}$
		(A1)	all correct.

Q	Solution	Mark	Notes
11	Suppose that $\sin x + \cos x \geq 1$ is not true. Then there exists an x in the given domain		
	for which $\sin x + \cos x < 1$	M1	
	$(\sin x + \cos x)^2 < 1^2$	A1	
	$\sin^2 x + 2\sin x\cos x + \cos^2 x < 1^2$		
	$1 + 2\sin x\cos x < 1$		
	$\sin x\cos x < 0$	A1	or $\sin 2x < 0$
	As $\sin x \geq 0$ and $\cos x \geq 0$, this is impossible.		
	Hence $\sin x + \cos x < 1$ cannot be true,		
	hence $\sin x + \cos x \geq 1$ for $0 \leq x \leq \dfrac{\pi}{2}$.	A1	cso

Q	Solution	Mark	Notes
12(a)(i)	f has an inverse function if and only if		
	f is both one-to-one (and onto).	B1	
12(a)(ii)	$ff^{-1}(x) = x$	B1	
12(b)(i)	g^{-1} exists if the domain of g is $[0, \infty)$	B1	or $(-\infty, 0]$ or subset of one of these
12(b)(ii)	Let $y = e^x + 1$	M1	
	$e^x = y - 1$		
	$x = \ln(y - 1)$		
	$h^{-1}(x) = \ln(x - 1)$	A1	
or			
	$h(x) = e^x + 1$		
	$x = e^{h^{-1}(x)} + 1$	(M1)	
	$e^{h^{-1}(x)} = x - 1$		
	$h^{-1}(x) = \ln(x - 1)$	(A1)	



		G1	$h(x)$ with $y=1$ as asymptote
		G1	$h^{-1}(x)$ with $x=1$ as asymptote
		G1	(0, 2), (2,0)

Q	Solution	Mark	Notes
12(b)(iii)	$gh(x) = g(e^x + 1)$	M1	accept $(h(x))^2 -1$
	$gh(x) = (e^x + 1)^2 - 1$		
	$gh(x) = e^{2x} + 2e^x$ or $e^x(e^x + 2)$	A1	

Q	Solution	Mark	Notes
13(a)	$R\sin(\theta - \alpha) \equiv 8\sin\theta - 15\cos\theta$		
	$R\sin\theta\cos\alpha$ - $R\cos\theta\sin\alpha \equiv 8\sin\theta - 15\cos\theta$	M1	oe si
	$R\cos\alpha = 8$		
	$R\sin\alpha = 15$		
	$R = \sqrt{8^2 + 15^2} = 17$	B1	
	$\alpha = \tan^{-1}\left(\frac{15}{8}\right) = 61.93°$	A1	
13(b)	$17\sin(\theta - 61.93°) = 7$		
	$\theta - 61.93° = \sin^{-1}\left(\frac{7}{17}\right)$	M1	
	$\theta - 61.93° = 24.32°, 155.68°$		
	$\theta = 86.24°$	A1	cao accept 86.25
	$\theta = 217.61°$	A1	cao
13(c)	$\frac{1}{8\sin\theta - 15\cos\theta + 23} = \frac{1}{17\sin(\theta - 61\cdot93^{\circ}) + 23}$		
	Greatest value = $\frac{1}{6}$	B1	
	Least value = $\frac{1}{40}$	B1	

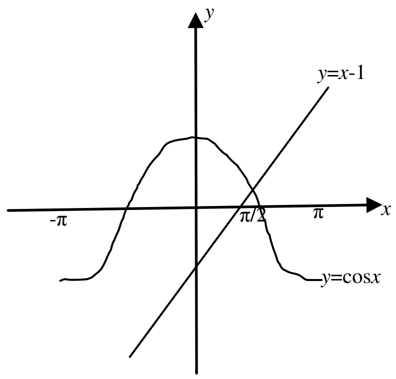
Q	Solution	Mark	Notes
14(a)	Use integration by parts	M1	
	$I = \left[\frac{x^4}{4}\ln x\right]_1^2 - \int_1^2 \frac{x^4}{4} \times \frac{1}{x}\,dx$	A1	1st term
		A1	2nd term
	$I = \left[\frac{x^4}{4}\ln x\right]_1^2 - \left[\frac{x^4}{16}\right]_1^2$	A1	2nd bracket
	$I = (4\ln 2) - (1 - \frac{1}{16})$	m1	correct use of limits
	$I = 4\ln 2 - \frac{15}{16} = 1.835$	A1	cao
14(b)	Let $x = 2\sin\theta$	M1	
	$dx = 2\cos\theta\, d\theta$		
	$\sqrt{4 - x^2} = \sqrt{4 - 4\sin^2\theta} = 2\cos\theta$		
	$x = 0, \theta = 0; x = 1, \theta = \frac{\pi}{6}$	A1	or 0, 1 if x used.
	$I = \int_0^{\frac{\pi}{6}} \frac{2 + 2\sin\theta}{2\cos\theta} 2\cos\theta dx$	A1	correct integrand
	$I = 2\int_0^{\frac{\pi}{6}} 1 + \sin\theta d\theta$		
	$I = 2[\theta - \cos\theta]_0^{\frac{\pi}{6}}$	A1	correct integration
	$I = 2(\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1)$	m1	correct use of limits
	$I = \frac{\pi}{3} + 2 - \sqrt{3} = 1.315$	A1	cao

Q	Solution	Mark	Notes
14(b)			
	Alternative solution		
	Let $x = 2\cos\theta$	(M1)	
	$dx = -2\sin\theta\, d\theta$		
	$\sqrt{4-x^2} = \sqrt{4-4cos^2\theta}$		
	$= 2\sqrt{1-cos^2\theta} = 2sin\theta$		
	$x = 0, \theta = \frac{\pi}{2}; x = 1, \theta = \frac{\pi}{3}$	(A1)	or 0, 1 if x used.
	$I = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{2+2cos\theta}{2sin\theta}(-2sin\theta)d\theta$	(A1)	correct integrand
	$I = -2\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 1 + cos\theta\, d\theta$		
	$I = -2[\theta + sin\theta]_{\frac{\pi}{2}}^{\frac{\pi}{3}}$	(A1)	correct integration
	$I = -2\left[\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)-\left(\frac{\pi}{2}+1\right)\right]$	(m1)	correct use of limits
	$I = -2\left(-\frac{\pi}{6}+\frac{\sqrt{3}}{2}-1\right)$		
	$I = \frac{\pi}{3} + 2 - \sqrt{3} = 1.315$	(A1)	cao

Q	Solution	Mark	Notes
15	$\int \frac{2\mathrm{d}y}{5-2y} = \int \mathrm{d}x$	M1	separate variable
			5-2y not separated.
	-ln|5 – 2y| = x (+ C)	A1	correct integration
	When x = 0, y = 1		
	-ln|3| = C	m1	use of boundary conditions
	ln|5 – 2y| - ln3= -x		
	$\frac{5-2y}{3}$ = e^{-x}	m1	inversion
	$y = \frac{1}{2}(5 - 3e^{-x})$	A1	cao any correct expression

Q	Solution	Mark	Notes
16(a)(i)			
	$\frac{dy}{dx} = e^{3\tan x} \times 3\sec^2 x$	M1	chain rule $e^{3\tan x}f(x)$
	$\frac{dy}{dx} = 3\sec^2 x\, e^{3\tan x}$	A1	$f(x)=3\sec^2 x$
16(a)(ii)			
	$\frac{dy}{dx} = \frac{x^2(\cos 2x \times 2) - \sin 2x(2x)}{x^4}$	M1	use of quotient rule oe
			$\frac{x^2 f(x) - \sin 2x g(x)}{x^4}$
	$\frac{dy}{dx} = \frac{2x\cos 2x - 2\sin 2x}{x^3}$	A1	$f(x)=2\cos 2x$ or $g(x)=2x$
		A1	cao
Alternative solution			
	$y = x^{-2}\sin 2x$		
	$\frac{dy}{dx} = -2x^{-3}\sin 2x + 2x^{-2}\cos 2x$	(M1)	use of product rule
			$f(x)\sin 2x + x^{-2}g(x)$
		(A1)	$f(x) = -2x^{-3}$ or $g(x) = 2\cos 2x$
		(A1)	cao

Q	Solution	Mark	Notes
16(b)	$3x^2\frac{dy}{dx} + 6xy + 2y\frac{dy}{dx} - 5 = 0$	B1	$3x^2\frac{dy}{dx}+6xy$
		B1	$2y\frac{dy}{dx}$
		B1	- 5
	$(3x^2 + 2y)\frac{dy}{dx} = 5 - 6xy$		
	$\frac{dy}{dx} = \frac{5-12}{3+4} = -1$	B1	cao
	Use of gradient = $-1/\frac{dy}{dx}$	M1	
	Equation of normal is $y - 2 = 1(x - 1)$		
	Equation of normal is $y = x + 1$	A1	correct equation any form

Q	Solution	Mark	Notes
17			
		G1	both graphs
	The two graphs intersect only once.	B1	
	(Root is between 0 and π/2.)		
	Using Newton-Raphson Method		
	$f(x) = x - 1 - \cos x$		
	$f'(x) = 1 + \sin x$	B1	or $-1 - \sin x$
	$x_{n+1} = x_n - \dfrac{x_n - 1 - \cos x_n}{1 + \sin x_n}$	M1	
	$x_0 = 1$		
	$x_1 = 1.293408$	A1	si
	$x_2 = 1.283436$		
	$x_3 = 1.283429$		
	$x_4 = 1.283429$		
	Root is 1.28 (correct to 2 d. p.)	A1	