GCE A LEVEL MARKING SCHEME

SUMMER 2018

A LEVEL (NEW)
MATHEMATICS – UNIT 4 APPLIED MATHEMATICS B
1300U40-1

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

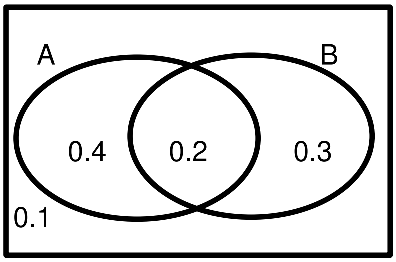
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

A2 Mathematics Unit 4: Applied Mathematics B

Solutions and Mark Scheme Summer 2018

<u>SECTION A – Statistics</u>

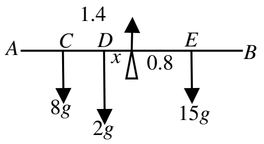
Qu. No.	Solution	Mark	Notes
1(a)	$1-P(A\cup B)$ $1-(P(A)+P(B)-P(A\cap B))=$ $1-(0.6+0.5-0.2)$ oe $=0.1$ (given)	M1 A1	M1 Use of $1-P(A\cup B)$ **ALTERNATIVE SOLUTION**  M1 for attempt at Venn diagram with one correct value. A1 for completely correct Venn diagram.
(b)	$P(only\ 1)=P(A)+P(B)-P(A\cap B)\times 2$ $P(only\ 1)=0.6+0.5-0.2\times 2$ $=0.7$ oe	 M1 A1	 M1 for 0.4 + 0.3
(c)	$P(B\|\text{not } A)=\dfrac{0.3}{0.4}$ $=\dfrac{3}{4}$ oe	B1 B1 B1 **[7]**	B1 for 0.3 as a numerator. FT for numerator only. B1 for 0.4 as a denominator. cao
2(a)	$p+p(1-p)=0.64$ oe $p^2-2p+0.64=0$ $(p-0.4)(p-1.6)=0$ $p=0.4$ or $p=1.6$ $p=0.4$	M1 A1 m1 A1	 Any correct method for solving 3 term quadratic equation. m0 if no working. A0 if 1.6 is not rejected.
(b)	$P(\text{Both field}\|\text{1st Male})=$ $\dfrac{9}{22}$ si $\times\dfrac{12}{32}$ si $=\dfrac{27}{176}=0.1534\ldots$	 M1 M1 A1 **[7]**	Alt Method $P(\text{Both field}\|\text{1st Male})=\dfrac{P(\text{male field }\cap\text{ field})}{\text{1st Male}}$ $\dfrac{\frac{9}{33}\times\frac{12}{32}}{\frac{22}{33}}$ M1 for $\frac{9}{33}\times\frac{12}{32}$ as a numerator M1 for $\frac{22}{33}$ as a denominator ISW

Qu. No.	Solution	Mark	Notes
3(a)(i)	(Continuous) uniform distribution and Parameters $(0,12)$	B1	
(ii)	Mean = 6 Variance = 12	B1 B1	SC1 for incorrect distribution with corresponding mean and variance.
(iii)	Valid assumption Eg. Assuming trains are running to time/not late/ any other equivalent	E1	
(b)(i)	$P(9 < X < 12) = 0.88 \times \frac{1}{4}$ $(= 0.22)$	M1	
	$P(12 < X < 19) = 0.12 \times \frac{7}{12}$ $(= 0.07)$	M1	
	$P(9 < X < 19) = 0.22 + 0.07$	m1	m1 for adding their 0.22 and 0.07. Dependent on either M1 awarded.
	$= 0.29$	A1	cao
(ii)	$P(1st\ train \mid waits\ between\ 9\ and\ 19\ mins)$		
	$= \frac{0.22}{0.29}$	m1	FT for both provided m1 awarded in (b)(i).
	$= \frac{22}{29} = 0.7586 \ldots$	A1	
		[10]	

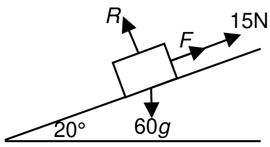
Qu. No.	Solution	Mark	Notes
4(a)	Valid reason e.g.(Approximately) symmetrical, taller in the middle, tails off at the ends, etc	E1	Do not accept bell curve alone.
(b)(i)	$P(60 \leq X \leq 70) = 0.26055$ Predicted number = 21	M1 A1	M1 for 0.26055 or 80×'their probability'. Allow 20.84 or '20 or 21' Use of tables gives $P(60 \leq X < 70) = 0.2618$ Predicted number is 21
(ii)	$P(X \geq 90) = 0.041518$ Predicted number = 3	M1 A1	M1 for 0.041518 or 80×'their probability'. Allow 3.32 or '3 or 4' Use of tables gives $P(X \geq 90) = 0.0418$ Predicted number is 3
(c)(i)	Valid comment e.g.18 is smaller than (predicted) 21 and 6 is bigger than (predicted) 3 so may not be the best model.	E1	Must include reference to model not being ideal/could be improved AND comparison of predicted values and actual values.
(ii)	Model could be improved by increasing the variance/standard deviation. (This would 'flatten out' the curve. It would lower the middle and lift up the tails.)	E1	
(d)	Valid comment. e.g. May not be suitable since the weekly household expenditure on food in Northern Ireland may have a different distribution. e.g. May be suitable as Northern Ireland is part of the UK and has a similar socioeconomic status to Wales.	E1	
		[8]	

5 (a)	p- value for 'Feed Wheat' versus 'Feed Oats' > 0.05 Price of Feed Wheat does not seem to be correlated with price of Feed Oats	E1 E1	
(b)	H_0: $\rho = 0$ H_1: $\rho \neq 0$ TS = 0.653 CV = 0.5760 Since TS>0.5760 Reject H_0. Sufficient evidence to suggest there is a correlation between the prices of feed wheat and wheat straw.	B1 B1 B1 B1 E1	H_0: $\rho = 0$ H_1: $\rho > 0$ TS = 0.653 CV = 0.4973 Since TS>0.4973 Reject H_0. Strong evidence to suggest there is a **positive** correlation between the prices of feed wheat and wheat straw.
(c)	Appropriate comment implying understanding that the second graph is comparing two wheat products whereas the first is comparing different grains.	E1 **[8]**	

SECTION B - Mechanics

Q	Solution	Mark	Notes
6			
	Moments about centre	M1	dim correct equation No missing forces
	$8g \times 1 \cdot 4 + 2g \times x = 15g \times 0 \cdot 8$	B1 A1	Any correct moment with pivot clearly indicated. correct equation
	$11 \cdot 2 + 2x = 12$ $2x = 0 \cdot 8$ $x = 0 \cdot 4$		
	$AD = 2 - 0 \cdot 4 = 1 \cdot 6$ (m)	A1	cao
		Total [4]	
	Alternative solution		
	Moments about A (or B, C, D, E) $8g \times 0 \cdot 6 + 2g \times x + 15g \times 2 \cdot 8 = 2R$	(M1) (B1)	No missing forces Any correct moment with pivot clearly indicated.
	$R = (8 + 2 + 15)g$ (= $25g = 245$)	(A1)	
	$4 \cdot 8 + 2x + 42 = 50$ $x = 1 \cdot 6$	(A1)	cao

Q	Solution	Mark	Notes
7(a)	$R = kv$		
	$0 \cdot 08 = 0 \cdot 2k$		
	$k = 0 \cdot 4$	B1	
	N2L applied to object, upwards positive	M1	dim correct eqn, all forces
	$0 - 0 \cdot 5g - R = 0 \cdot 5a, \qquad a = \frac{\mathrm{d}v}{\mathrm{d}t}$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = -9 \cdot 8 - 0 \cdot 8v$	A1	convincing
		(3)	
7(b)	$\int \frac{\mathrm{d}v}{9{\cdot}8+0{\cdot}8v} = -\int \mathrm{d}t$	M1	separating variables and an attempt to integrate
	$\frac{1}{0{\cdot}8}\ln\lvert 9 \cdot 8 + 0 \cdot 8v\rvert = -t + (C)$	A1	
	when $t = 0, v = 24$	m1	used
	$C = \frac{1}{0{\cdot}8}\ln\lvert 29\rvert$	A1	
	$-0 \cdot 8t = \ln\left\lvert\frac{9{\cdot}8+0{\cdot}8v}{29}\right\rvert$		
	$29e^{-0{\cdot}8t} = 9 \cdot 8 + 0 \cdot 8v$	m1	inversion
	$0 \cdot 8v = 29e^{-0{\cdot}8t} - 9 \cdot 8$		
	$v = 36 \cdot 25e^{-0{\cdot}8t} - 12 \cdot 25$	A1	cao
		(6)	
7(c)	At highest point, $v = 0$	M1	used
	$t = 1 \cdot 25(\ln\lvert 29\rvert - \ln\lvert 9 \cdot 8\rvert)$		
	$t = 1 \cdot 356$ (s)	A1	cao
		(2)	
		Total [11]	

Q	Solution	Mark	Notes
8(a)			
	$R = 60g\cos 20°$	B1	si
	$F = 0.3 \times R \quad (= 165 \cdot 761 \ldots)$	B1	si $\quad (F = 18g\cos 20°)$
	Apply N2L to object, downwards +ve	M1	dim correct equ, all forces
	$60g\sin 20° - F - 15 = 60a$	A2	-1 each error
	$a = 0 \cdot 3391 \quad (\text{ms}^{-2})$	A1 **(6)**	cao
8(b)	Resultant tractive force up plane	M1	dim correct, all forces
	$= 350 - 60g\sin 20° = 148 \cdot 892$	A1	
	Limiting friction $= 165 \cdot 761 \ldots$		
	Resultant tractive force < Limiting friction		
	Object **does not** move up the plane.	A1 **(3)**	convincing
		Total [9]	

Alternative solution to (b)

	(Maximum) force that can be applied up the slope (without object slipping)	(M1)	dim correct, all forces
	$= 60g\sin 20° + 0.3 \times 60g\cos 20° = 366 \cdot 86 \ldots$	(A1)	
	$T = 350 < 366 \cdot 86 \ldots$		
	Object **does not** move up the plane.	(A1)	convincing argument with reference to max/limiting friction

Q	Solution	Mark	Notes
9(a)	For *P*,		
	initial horizontal velocity $= 24\cdot 5\cos 30°$	B1	si
	$= 12\cdot 25\sqrt{3}$		
	Initial vertical velocity $= 24\cdot 5\sin 30°$	B1	si
	$= 12\cdot 25$		
	For time of flight, use $s = ut + \frac{1}{2}at^2$ with		
	$s = 0,\ a = (\pm)9\cdot 8, u = (\pm)12\cdot 25$	M1	ft u_{vert}
	$0 = 12\cdot 25t - 4\cdot 9t^2$	A1	oe, ft u_{vert} provided direction opposes g
	$t = 2\cdot 5$		
	Range, $R = 12\cdot 25\sqrt{3} \times 2\cdot 5 = 30\cdot 623\sqrt{3}$		
	$= 53(\cdot 044)$ (m)	A1	cao
		(5)	
9(b)	Horizontal distance travelled by *P* D_P		
	$= 12\cdot 25\sqrt{3} \times t$		
	Horizontal distance travelled by *Q* D_Q		
	$= 12\cdot 25\sqrt{3} \times (t-1)$	B1	both distances ft u_{horiz} from (a)
	$D_P + D_Q = R$ $D_P + D_Q = 30\cdot 623\sqrt{3}$	M1	ft R and u_{horiz} from (a)
	$12\cdot 25\sqrt{3} \times t\ +$ $+ 12\cdot 25\sqrt{3} \times (t-1) = 30\cdot 623\sqrt{3}$		
	$t = 1\cdot 75$	A1	cao
OR	$H_P = 12\cdot 25t - 4\cdot 9t^2$		
	$H_Q = 12\cdot 25(t-1) - 4\cdot 9(t-1)^2$	(B1)	both distances ft u_{vert} from (a)
	Collision occurs when $H_P = H_Q$	(M1)	used. ft u_{vert} from (a)
	$t = 1\cdot 75$	(A1)	cao
	For height, use $s = ut + \frac{1}{2}at^2$ with		
	$a = -9\cdot 8, u = 12\cdot 25, t = 1\cdot 75$	M1	u must oppose g $(t - 1 = 0\cdot 75)$ ft $t, t-1\ \ (t > 1)$
	$s = (12\cdot 25)(1\cdot 75) + \frac{1}{2}(-9\cdot 8)(1\cdot 75)^2$		
	Height, $H = 6\cdot 4\,(3125)$ (m)	A1	cao
		(5)	
		Total [10]	

Q	Solution	Mark	Notes
10(a)	$\mathbf{F} = m\mathbf{a}$	M1	used
	$\mathbf{a} = -\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}$		
	$\lvert\mathbf{a}\rvert = \sqrt{\left(-\frac{3}{2}\right)^2 + (2)^2 + \left(-\frac{5}{2}\right)^2}$	m1	
	$\lvert\mathbf{a}\rvert = \frac{5\sqrt{2}}{2} = 3 \cdot 54$ (ms^{-2})	A1 **(3)**	cao
	Alternative solution to (a)		
	$\lvert\mathbf{F}\rvert = \sqrt{(-3)^2 + 4^2 + (-5)^2} = \sqrt{50}$	(M1)	
	$F = ma$	(m1)	used
	$\lvert\mathbf{a}\rvert = \frac{5\sqrt{2}}{2} = 3 \cdot 54$ (ms^{-2})	(A1)	cao
10(b)	Use $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $(+\mathbf{r_0})$		
	with $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{a} = -\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}$	M1	
	$\mathbf{r} = (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times 2 + \frac{1}{2}\left(-\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}\right) \times 2^2$	A1	
	$\mathbf{r} = (6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) + (-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$		
	$\mathbf{r} = 3\mathbf{i} - 3\mathbf{k}$		
	position vector $= (3\mathbf{i} - 3\mathbf{k}) + (2\mathbf{i} - 7\mathbf{j} + 9\mathbf{k})$		
	position vector $= 5\mathbf{i} - 7\mathbf{j} + 6\mathbf{k}$	A1 **(3)**	cao
		Total [6]	
	Alternative solution to (b)		
	$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \left(-\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}\right)t + \mathbf{v_0}$		
	$= \left(-\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}\right)t + (3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$		
	$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t = \left(-\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}\right)\frac{t^2}{2}$ $+(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})t + (\mathbf{r_0})$	(M1)	attempt to integrate twice $\mathbf{v_0}$ must be present in $\mathbf{v}$
	$= \left(-\frac{3}{2}\mathbf{i} + 2\mathbf{j} - \frac{5}{2}\mathbf{k}\right)\frac{t^2}{2} + (3\mathbf{i} - 2\mathbf{j} + \mathbf{k})t$ $+(2\mathbf{i} - 7\mathbf{j} + 9\mathbf{k})$	(A1)	oe
	At $t = 2$, $\mathbf{r} = 5\mathbf{i} - 7\mathbf{j} + 6\mathbf{k}$	(A1)	cao