GCE AS MARKING SCHEME

SUMMER 2019

AS (NEW)
MATHEMATICS
UNIT 1 PURE MATHEMATICS A
2300U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS

AS UNIT 1 PURE MATHEMATICS A

SUMMER 2019 MARK SCHEME

Q	Solution	Mark	Notes
1	$3\frac{\sin\theta}{\cos\theta} + 2\cos\theta = 0$	M1	use of $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$
	$3\sin\theta + 2\cos^2\theta = 0$		
	$3\sin\theta + 2(1 - \sin^2\theta) = 0$	M1	use of $\sin^2\theta + \cos^2\theta \equiv 1$
	$2\sin^2\theta - 3\sin\theta - 2 = 0$		
	$(2\sin\theta + 1)(\sin\theta - 2) = 0$	m1	oe coeff sin multiply to give coeff $\sin^2$; constant terms multiply to give constant term
	Note No working shown m0		
	$\sin\theta = 2$ (no solution)		
	$\sin\theta = -\frac{1}{2}$	A1	cao
	$\theta = 210°, 330°$	B1B1	ft sin +ve for B1 ft sin –ve for B1B1
	-1 each additional incorrect answer in range up to -2.		
	Ignore answers outside range.		

Notes

If both branches give valid solutions:

+ve, +ve	mark the correct branch for B1
-ve, -ve	mark the branch that give most marks for B1 B1
+ve, -ve	mark +ve for B1; mark -ve for B1 B1 and award the marks for the branch that gives the most marks.

If both branches do not give solutions B0 B0.

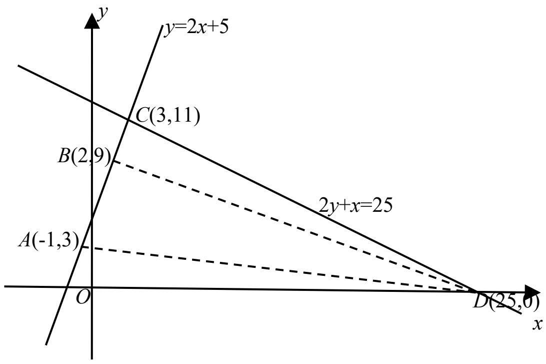
Q	Solution	Mark	Notes
2	$x^2 + (2k + 4)\,x + 9k = 0$	B1	terms grouped, brackets not required, si
	Discriminant = $(2k + 4)^2 - 4\times1\times9k$	B1	An expression for $b^2 - 4ac$ with at least two of a, b or c correct
	Discriminant = $4k^2 - 20k + 16$	B1	cao
	If distinct real roots, discriminant> 0	M1	allow $\geq$ May be implied by later work
	$k^2 - 5k + 4 > 0$		
	$(k - 1)(k - 4) > 0$		
	Critical values, $k = 1, 4$	B1	ft if quadratic has 3 terms
	$k < 1$ or $k > 4$	A2	ft their critical values if quadratic has 3 terms
		(A1)	non strict inequalities.
		(A1)	'and' not 'or' used.
		(A1)	$1 > k > 4$.

Q	Solution	Mark	Notes
3	Let $f(x) = 12x^3 - 29x^2 + 7x + 6$		
	$f(1) = 12 - 29 + 7 + 6 \neq 0$	M1	correct use of factor theorem
	$f(2) = 96 - 116 + 14 + 6 = 0$		
	so $(x - 2)$ is a factor.	A1	
	$f(x) = (x - 2)(12x^2 + ax + b)$	M1	ft their linear factor a or b correct
	$f(x) = (x - 2)(12x^2 - 5x - 3)$	A1	cao
	$f(x) = (x - 2)(3x + 1)(4x - 3)$	m1	coeffs of x multiply to their 12 constant terms multiply to their -3 or formula with correct a, b, c.
	When $f(x) = 0$, $x = 2, -\frac{1}{3}, \frac{3}{4}$	A1	cao

<u>Note</u>

Answers only with no working 0 marks.

Q	Solution	Mark	Notes
4			



Q	Solution	Mark	Notes
4(a)	Gradient L_1 = grad AB = $\frac{9-3}{2-(-1)} = \frac{6}{3} = 2$	B1	
	correct method for finding eq^n of line	M1	
	Eq^n of L_1 is $y - 3 = 2(x + 1)$	A1	$y - 9 = 2(x - 2)$ convincing
	$y = 2x + 5$		
4(b)(i)	$2y + x = 25$		
	When $y = 0$, $x = 25$	B1	isw
	D has coordinates $D(25, 0)$		
4(b)(ii)	Gradient of $L_2 = -\frac{1}{2}$	B1	
	grad L_1×grad $L_2 = 2\times-\frac{1}{2} = -1$		
	Therefore L_1 and L_2 are perpendicular	B1	Statement required
4(b)(iii)	$2y = 4x + 10$		
	$2y = -x + 25$		
	Solving simultaneously	M1	one variable eliminated. Some working required
	$x = 3$, $y = 11$	A1	cao

Q	Solution	Mark	Notes
4(c)	length $CD = \sqrt{(3-25)^2 + (11-0)^2}$	M1	FT their C and D
	$= \sqrt{605} = (11\sqrt{5}) = (24.6)$	A1	cao
4(d)	length $AC = \sqrt{4^2 + 8^2} (= 4\sqrt{5})$	B1	ft C
	length $BC = \sqrt{2^2 + 1^2} (= \sqrt{5})$	B1	ft C
	$\tan \angle ADC = \dfrac{AC}{DC}$ or $\tan \angle BDC = \dfrac{BC}{DC}$	M1	method for relevant angle
	$\angle ADC = \tan^{-1}\left(\dfrac{4\sqrt{5}}{11\sqrt{5}}\right) (= 19.983°)$		
	$\angle BDC = \tan^{-1}\left(\dfrac{\sqrt{5}}{11\sqrt{5}}\right) (= 5.194°)$	A1	a correct angle ft lengths
	$\angle ADB = 19.983° - 5.194° = 14.8°$	A1	cao
OR			
	Length $AB = \sqrt{45} = 3\sqrt{5}$		
	Length $DB = \sqrt{610}$		
	Length $AD = \sqrt{685}$	(B1)	any correct length ft D
		(B1)	all 3 correct ft D
	$\cos ADB = \dfrac{610+685-45}{2(\sqrt{610})(\sqrt{685})}$	(M1)	attempt at cosine rule, 1 slip only.
		(A1)	correct cosine rule, ft lengths
	Angle $ADB = 14.8°$	(A1)	cao

Q	Solution		Mark	Notes
5	Using proof by exhaustion		M1	attempt to find $2n^2 + 5$ for n=1,2,3,4
	n	$2n^2 + 5$		
	1	7		
	2	13		
	3	23		
	4	37	B1	at least 3 correct
	7, 13, 23 and 37 are prime numbers.		A1	
	Therefore the statement is true.			

Q	Solution	Mark	Notes
6(a)(i)	**AC** = -**a** + **c**	B1	
6(a)(ii)	**OD** = **a** + $\frac{1}{2}$**c**	B1	
6(a)(iii)	**OE** = **c** + $\frac{2}{3}$**a**	B1	any correct form
6(b)	Valid reason	M1	ft (a)
	Eg. **DE** ≠ *k***AC**, **DE** = $\frac{1}{2}$**c** – $\frac{1}{3}$**a**;		
	or *E* is not the midpoint of *CB*.		
	Hence *AC* is not parallel to *DE*.	A1	ft (a)

Q	Solution	Mark	Notes
7(a)	$\dfrac{(2\sqrt{3}+a)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$	M1	
	$= \frac{1}{(3-1)}(2\times 3 + 2\sqrt{3} + a\sqrt{3} + a\times 1)$	A1	numerator correct
	$= \dfrac{1}{2}(6 + 2\sqrt{3} + a\sqrt{3} + a)$	A1	all correct isw
	$(= \dfrac{1}{2}(6+a) + \dfrac{1}{2}(2+a)\ \sqrt{3}\ )$		
7(b)	$\dfrac{2b\sqrt{2}\sqrt{3}}{\sqrt{2}}$ - $3\sqrt{3}$ + $8\sqrt{3}$	B1	for $\frac{2b\sqrt{2}\sqrt{3}}{\sqrt{2}}$ or $\frac{2b\sqrt{6}}{\sqrt{2}}$ or $\sqrt{12b^2}$ or $2b\sqrt{3}$ or $2\sqrt{3b^2}$
		B1	for $\pm 3\sqrt{3}$ and $\pm 8\sqrt{3}$ or $5\sqrt{3}$
	$= 2b\sqrt{3} + 5\sqrt{3} = (2b+5)\sqrt{3}$	B1	cao

Note

Mark final answer

Q	Solution	Mark	Notes
8(a)	$y + \delta y = 2(x + \delta x)^2 - 5(x + \delta x)$	B1	
	$y + \delta y = 2x^2 + 4x\delta x + 2(\delta x)^2 - 5x - 5\delta x$		
	Subtract $y = 2x^2 - 5x$ from $y + \delta y$	M1	
	$\delta y = 4x\delta x - 5\delta x + 2(\delta x)^2$	A1	
	$\frac{\delta y}{\delta x} = 4x - 5 + 2(\delta x)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{Lim}_{\delta x \to 0} \frac{\delta y}{\delta x}$	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 5$	A1	All correct
	OR		
	$\mathrm{f}(x + h) = 2(x + h)^2 - 5(x + h)$	(B1)	
	$\mathrm{f}(x + h) = 2x^2 + 4xh + 2h^2 - 5x - 5h$		
	$\mathrm{f}(x + h) - \mathrm{f}(x) = 4xh - 5h + 2h^2$	(M1A1)	
	$\frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h} = 4x - 5 + 2h$		
	$\mathrm{f}'(x) = \mathrm{Lim}_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$	(M1)	
	$\mathrm{f}'(x) = 4x - 5$	(A1)	All correct

Q	Solution	Mark	Notes
8(b)	$y = \frac{16}{5}x^{\frac{1}{4}} + 48x^{-1}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16}{5} \times \frac{1}{4}x^{-\frac{3}{4}} - 48x^{-2}$	B1	one correct term
		B1	second correct term
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{5}x^{-\frac{3}{4}} - 48x^{-2}$		
	When $x = 16$,		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{5}(16)^{-\frac{3}{4}} - 48(16)^{-2}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{7}{80} = (-0.0875)$	B1	cao

Q	Solution	Mark	Notes
9(a)	Centre of circle is $\left(\frac{6+(-2)}{2}, \frac{4+10}{2}\right)$		
	$= (2, 7)$	B1	convincing
9(b)	Radius = $\sqrt{3^2+4^2} = 5$	B1	accept radius^2
	Eq^n of circle is $(x-2)^2+(y-7)^2 = 5^2$	M1	ft radius
	Eq^n of circle is $x^2+y^2-4x-14y+28=0$	A1	cao
OR	Radius = $r = \sqrt{3^2+4^2} = 5$	(B1)	accept radius^2
	Eq^n of circle is $x^2+y^2-4x-14y+c=0$	(M1)	implied by a=-4, b=-14.
	$c = f^2+g^2-r^2 = 2^2+7^2-5^2 = 28$	(A1)	
OR	Eq^n of circle is $(x-2)^2+(y-7)^2 = k$	(B1)	
	Eq^n of circle is $x^2+y^2-4x-14y+c=0$	(M1)	implied by a=-4, b=-14.
	At (-2, 4) $2^2+4^2-4\times(-2)-14\times4+c=0$		
	$c = 28$	(A1)	
9(c)	Solve eq^{ns} simultaneously	M1	
	$x^2-3x-10=0$	A1	ft equation of circle, oe
	$(x+2)(x-5)=0$	m1	correct method for solving quadratic
	$x = 5$	A1	cao
	$y = 11$	A1	ft x
	$C(5, 11)$		

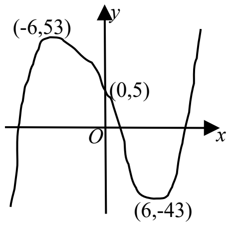
Q	Solution	Mark	Notes
9(d)	Area $ABC = \frac{1}{2} \times AC \times BC$	M1	oe
	$AC = 7\sqrt{2}$, $BC = \sqrt{2}$	m1	correct method for a distance ft C
	Area $ABC = 7$	A1	cao

Q	Solution	Mark	Notes
10(a)	$3^{3x}.3^{2y} = 3^3$	M1	oe
	$3x + 2y = 3$	A1	
	$2^{-3x}.2^{-3y} = 2^{-6}$	M1	oe
	$3x + 3y = 6$	A1	
	$x = -1$	A1	cao
	$y = 3$	A1	cao
10(b)	$2\log_a x = \log_a x^2$	B1	use of power law
	$\log_a(5x+2)+\log_a(x-1) = \log_a(5x+2)(x-1)$	B1	use of add/subtraction law on any two log terms
	correct elimination of logs	M1	
	$3x^2 = (5x + 2)(x - 1)$	A1	oe cao
	$3x^2 = 5x^2 - 3x - 2$		
	$2x^2 - 3x - 2 = 0$		
	$(2x + 1)(x - 2) = 0$	m1	coeffs x multiply to their 2 and constant terms multiply to their -2. Or formula
	Note No method shown m0		
	$x = -\frac{1}{2}$ or $x = 2$	A1	cao
	($x \neq -\frac{1}{2}$ since $\log_a x = \log_a(-\frac{1}{2})$ is undefined)		
	therefore $x = 2$	B1	ft solutions if one +ve, one -ve

Q	Solution	Mark	Notes
11	Attempt to take logs	M1	
	$\log_{10}Q = 3\log_{10}P + \log_{10}1.25$	A1	
	This is the equation of a straight line of the form $y = mx + c$.		
	gradient = 3	B1	
	intercept = $\log_{10}1.25$ (= 0.09691)	B1	

Q	Solution	Mark	Notes
12(a)	9	B1	
12(b)	4^{th} term = ${}^8C_3(2)^5(-5x)^3$	M1	si condone 5
	$= -224000x^3$	A1	
12(c)	The greatest coefficient is in the 7^{th} term	B1	si, oe
	Attempt to find 3^{rd} or 5^{th} or 7^{th} or 9^{th} term	M1	
	Greatest coefficient = ${}^8C_6(2)^2(-5)^6$		
	Greatest coefficient = 1750000	A1	condone x^6

Q	Solution	Mark	Notes
13(a)	$\frac{dy}{dx} = \frac{1}{3}x^2 - k$	B1	
	When $x = 3$, $\frac{dy}{dx} = -9$	M1	used
	$\frac{1}{3} \times 3^2 - k = -9$		
	$k = 12$	A1	convincing, allow verification
13(b)	At stationary points $\frac{dy}{dx} = 0$	M1	used
	$\frac{1}{3}x^2 - 12 = 0, \quad x^2 = 36$		
	$x = -6, 6$	A1	one correct pair
	$y = 53, -43$	A1	second correct pair
	Note: Allow y values shown in (c) but do not ft for incorrect x values.		
	$\frac{d^2y}{dx^2} = \frac{2}{3}x$	M1	oe
	When $x = -6$, $\frac{d^2y}{dx^2} = -4 < 0$		
	(-6, (53)) is a maximum point	A1	ft x
	When $x = 6$, $\frac{d^2y}{dx^2} = 4 > 0$		
	(6, (-43)) is a minimum point	A1	ft x provided different conclusion

Q	Solution	Mark	Notes
13(c)		M1 A1	+ve cubic curve points, ft if possible

Q	Solution	Mark	Notes
14	Area of triangle = $\frac{1}{2}bc\sin A$	M1	used
	$14 = \frac{1}{2} \times 5 \times x \sin 120°$	A1	
	$x = \frac{56\sqrt{3}}{15} = 6.47$	A1	either form, accept [6.4, 6.5]
	use cosine rule	M1	allow 1 slip
	$y^2 = 5^2 + x^2 - 2 \times 5 \times x \cos 120°$		
	$y^2 = 5^2 + 6.47^2 - 2 \times 5 \times 6.47 \times \cos 120°$	A1	ft x
	$y = 9.96$	A1	cao, 2 dp required

Q	Solution	Mark	Notes
15	$f'(x) = 3x^2 - 12x + 13$	M1	attempt to differentiate
	$f'(x) = 3(x-2)^2 - 12 + 13$	m1	
	$f'(x) = 3(x-2)^2 + 1$	A1	
	Hence $f'(x) > 0$ for all values of x,	E1	depends on previous A1
	and $f(x)$ is an increasing function.		Accept $\geq$
OR			
	$f'(x) = 3x^2 - 12x + 13$	(M1)	
	Discriminant = $(-12)^2 - 4\times3\times13 = -12 < 0$	(m1)	
	So $f'(x)$ does not cross the x-axis		
	$f'(1) = 3 - 12 + 13 = 4 > 0$	(A1)	oe
	Hence $f'(x) > 0$ for all values of x,	(E1)	depends on previous A1.
	and $f(x)$ is an increasing function.		Accept $\geq$

Q	Solution	Mark	Notes
16	Curve cuts the x-axis when x = -2, -1 and 2	B1	implied by limits
	$y = x^3 + x^2 - 4x - 4$	B1	
	$I_1 = \int_{-2}^{-1}(x^3 + x^2 - 4x - 4)\,dx$	M1	attempt to integrate y wrt x
			Limits not required.
			At least one power of x increased
	$= \left[\frac{x^4}{4} + \frac{x^3}{3} - 2x^2 - 4x\right]_{-2}^{-1}$	A1	correct integration,
			ft provided cubic
	$= \left[\frac{23}{12} - \frac{4}{3}\right]$	m1	correct use of limits,
			implied by 7/12
	$= \frac{7}{12}$	A1	cao
	Note Must be supported by workings.		
	$I_2 = \int_{-1}^{2}(x^3 + x^2 - 4x - 4)\,dx$	(M1 if not previously awarded)	
	$= \left[\frac{x^4}{4} + \frac{x^3}{3} - 2x^2 - 4x\right]_{-1}^{2}$	(A1 if not previously awarded)	
	$= \left[-\frac{28}{3} - \frac{23}{12}\right]$	(m1 if not previously awarded)	
	$= -\frac{45}{4}$	(A1 cao if not previously awarded)	
	Total area = $\frac{7}{12} + \frac{45}{4}$	m1	ft areas
	Total area = $\frac{71}{6}$ (= 11.833)	A1	cso