



GCE A LEVEL MARKING SCHEME

SUMMER 2019

**A LEVEL (NEW)
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1**

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS
A2 UNIT 3 PURE MATHEMATICS B
SUMMER 2019 MARK SCHEME

Q	Solution	Mark	Notes
1(a)	$\frac{9}{(x-1)(x+2)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$ $9 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$ Put $x = 1, A = 1$ Coef. $x^2, 0 = A + B, B = -1$ Put $x = -2, C = -3$	M1 m1 m1 A1	 correct method to remove denominator correct method for finding A, B or C all 3 values correct, cao
1(b)	$\int \frac{9}{(x-1)(x+2)^2} dx$ $= \int \frac{1}{(x-1)} dx - \int \frac{1}{(x+2)} dx - \int \frac{3}{(x+2)^2} dx$ $= \ln x-1 - \ln x+2 + \frac{3}{(x+2)} + Const$	M1 A1 A1	attempt to integrate PF one correct term, ft (a) all correct, -1 if no <i>Const.</i> ft(a), isw

Note: ft incorrect constants or $\frac{A}{(x-1)} + \frac{Bx+C}{(x+2)^2}$.

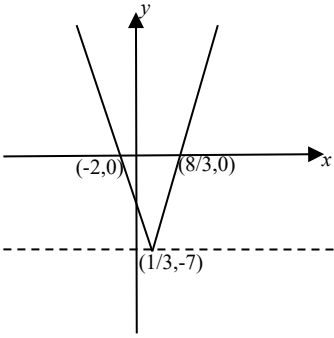
Q	Solution	Mark	Notes
2	$(4-x)(1+2x)^{-\frac{1}{2}}$ $= (4-x) \left[1 + \left(-\frac{1}{2}\right)(2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(2x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3 \times 2}(2x)^3 \right]$	B2	-1 each error term (4-x) not required.
	$= (4-x) \left[1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \dots \right]$		
	$= (4 - 4x + 6x^2 - 10x^3) - (x - x^2 + \frac{3}{2}x^3) + \dots$	M1	correct method
	$= 4 - 5x + 7x^2 - \frac{23}{2}x^3 + \dots$	A2	terms all correct
			-1 each incorrect term. Ignore further terms., isw
	Expansion is valid for $ x < \frac{1}{2}$.	B1	oe, mark final answer
<u>Note</u>	$(1+2x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(2x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(2x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}(2x)^3 + \dots$ $= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots$	(B0)	
	$(4-x)(1+2x)^{\frac{1}{2}} = 4 + 3x - 3x^2 + \frac{5}{2}x^3 + \dots$	(M1)	correct method
		(A2)	terms all correct
			-1 each incorrect term. Ignore further terms.
	Expansion is valid for $ x < \frac{1}{2}$.	(B1)	

Q	Solution	Mark	Notes
3(a)	$x_2 = 29$	B1	
	$x_1 = 8$	B1	ft 1 slip
3(b)	Not an AP because $113 - 29 \neq 29 - 8$		
	$x_3 - x_2 \neq x_2 - x_1.$	B1	either GP or AP statement, reason required, ft (a)
	Not a GP because $\frac{113}{29} \neq \frac{29}{8}$		
	$\frac{x_3}{x_2} \neq \frac{x_2}{x_1}.$	B1	for the other statement, Reason required, ft (a)

Note : Accept equivalent statement if clear.

Q	Solution	Mark	Notes
4(a)	$5\sin x - 12\cos x = R\sin x \cos \alpha - R\cos x \sin \alpha$ $R\cos \alpha = 5$ $R\sin \alpha = 12$ $R = \sqrt{5^2 + 12^2} = 13$ $\alpha = \tan^{-1}\left(\frac{12}{5}\right) = 67.380^\circ$	M1 B1 A1	accept 1.176 rad not 1.176.
4(b)	$y = \frac{4}{13\sin(x - 67.380) + 15}$ Min y when denominator is max, ie when $\sin(x - 67.380) = 1$ Min $y = \frac{4}{28}$ ($= \frac{1}{7} = 0.1429$)	M1 A1	implied by correct min ft R
4(c)	$\sin(x - 67.380) = -\frac{3}{13}$ $x - 67.380 = -13.342, 193.342$ $x = 54.04^\circ$ $x = 260.72^\circ$	M1 A1 A1 A1	either value 0.943 rad 4.550 rad Ignore answers outside the range. -1 for any extra answers within range Accept answers rounding correctly to 54, 261.

Note: full follow through their R and α provided of equivalent difficulty, eg not $\alpha = 0, R = 1$.

Q	Solution	Mark	Notes
5(a)	either $1 - 3x > 7$, or $1 - 3x < -7$	M1	one correct inequality
		A1	both correct
	either $x < -2$, or $x > \frac{8}{3}$	A1	cao oe, mark final answer
OR			
	$(1 - 3x)^2 > 7^2$	(M1)	
	$9x^2 - 6x - 48 > 0$	(A1)	
	$3x^2 - 2x - 16 > 0$		
	$(3x - 8)(x + 2) > 0$		
	either $x < -2$, or $x > \frac{8}{3}$	(A1)	cao oe
5(b)			
		G1	Shape, min below x-axis
		G1	$(-2, 0)$, $(8/3, 0)$, ft (a)
		B2	$(1/3, -7)$ and drawn in correct quadrant.
			B1 for either coordinate.

Q	Solution	Mark	Notes
6(a)	$\frac{dx}{d\theta} = \cos \theta$	B1	si
	$\frac{dy}{d\theta} = -2 \sin 2\theta$	B1	si
	$\frac{dy}{dx} = -\frac{2 \sin 2\theta}{\cos \theta}$	M1	
	When $\theta = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}, y = 0$	B1	si
	$\frac{dy}{dx} = -\frac{2 \sin \frac{\pi}{2}}{\cos \frac{\pi}{4}} = -2\sqrt{2}$	A1	$m = -2\sqrt{2}$, oe
			ft $dy/d\theta = 2\sin 2\theta$ or $-\sin 2\theta$
	Eq ⁿ of tgt is $y - 0 = -2\sqrt{2} \left(x - \frac{1}{\sqrt{2}}\right)$		
	Eq ⁿ of tgt is $y = -2\sqrt{2}x + 2$	A1	$c = 2$ cao
OR	$y = \cos 2\theta = 1 - 2 \sin^2 \theta$	(M1)	
	$y = 1 - 2x^2$	(A1)	
	$\frac{dy}{dx} = -4x$	(B1)	ft $2 \sin^2 \theta - 1$
	When $\theta = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}, y = 0$	(B1)	si
	$\frac{dy}{dx} = -4 \times \frac{1}{\sqrt{2}} = -2\sqrt{2}$	(B1)	ft $2 \sin^2 \theta - 1$
	Eq ⁿ of tgt is $y - 0 = -2\sqrt{2} \left(x - \frac{1}{\sqrt{2}}\right)$		
	Eq ⁿ of tgt is $y = -2\sqrt{2}x + 2$	(A1)	$c = 2$, cao

Q	Solution	Mark	Notes
6(b)	$x + y = 1$		
	$\cos 2\theta + \sin \theta - 1 = 0$	M1	
	$1 - 2 \sin^2 \theta + \sin \theta - 1 = 0$	M1	$\cos 2\theta = 1 - 2 \sin^2 \theta$
	$2 \sin^2 \theta - \sin \theta = 0$		
	$\sin \theta (2 \sin \theta - 1) = 0$	m1	si, ft for correct factorisation
	$x = \sin \theta = 0, \frac{1}{2}$	A1	one correct pair cao
	$y = 1 - x = 1, \frac{1}{2}$	A1	all correct cao
	required coordinates are $(0, 1), \left(\frac{1}{2}, \frac{1}{2}\right)$		

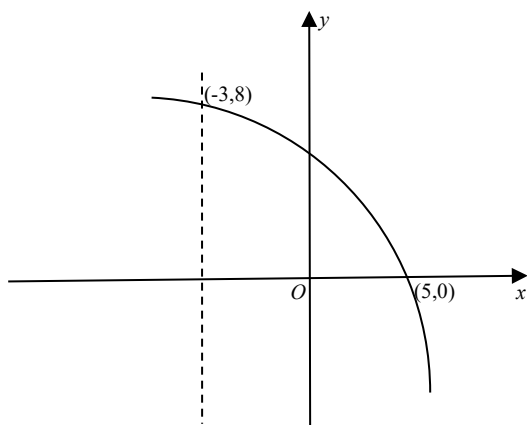
OR

$y = \cos 2\theta = 1 - 2 \sin^2 \theta$	(M1)	$\cos 2\theta = 1 - 2 \sin^2 \theta$
$y = 1 - 2x^2$	(m1)	$x = \sin \theta$
$y = 1 - x$		
Solving simultaneously	(m1)	
$x(2x - 1) = 0$		
$x = 0, x = \frac{1}{2}$	(A1)	one correct pair
$y = 0, y = \frac{1}{2}$	(A1)	all correct
required coordinates are $(0, 1), \left(\frac{1}{2}, \frac{1}{2}\right)$		

Q Solution

Mark Notes

7(a)

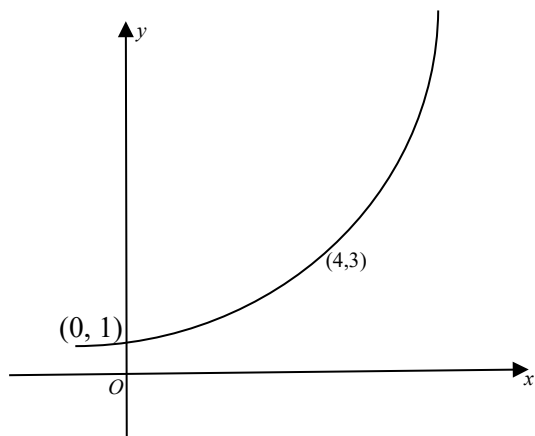


G1 shape of graph

B1 $(5, 0)$

B1 $(-3, 8)$

7(b)



G1 shape, intersecting y-axis at a positive value of y .

B1 $(4, 3)$

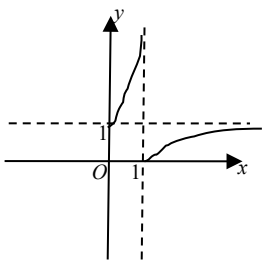
$(0, 1)$ not required.

Q	Solution	Mark	Notes
8(a)	$T_3 = 3 + 2d$ $T_{19} = 3 + 18d$ $T_{67} = 3 + 66d$ $\frac{3+66d}{3+18d} = \frac{3+18d}{3+2d} (=r)$ $(3+66d)(3+2d) = (3+18d)(3+18d)$ $9 + 204d + 132d^2 = 9 + 108d + 324d^2$ $192d^2 = 96d$ $d = \frac{1}{2}$	B1 B1 M1 m1 A1	T ₃ , T ₁₉ or T ₆₇ correct all correct method for d or r method for d cao, condone presence of $d = 0$
8(b)(i)	AP $a = 100$, $d = 12$ 8 weeks = 40 working days. Total no. employees = $100 + 39 \times 12$ Total no. employees = 568	M1 m1 A1	si
8(b)(ii)	Wage bill = $55[100 + 112 + 124 + \dots(40 \text{ terms})]$ Wage bill = $55 \left[\frac{40}{2} (2 \times 100 + 39 \times 12) \right]$ Wage bill = $55 \left[\frac{40}{2} (100 + 568) \right]$ Wage bill = (£)734 800	M1 m1 (m1) A1	55 not required, implied by 13360 ft (b)(i) cao

Q	Solution	Mark	Notes
9(a)	$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - 2 \cot \beta \tan \beta}$ $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - 2}$ $\tan(\alpha + \beta) = -(\tan \alpha + \tan \beta)$	M1 A1	$\tan \alpha \tan \beta = 2$ convincing
9(b)	$4 \tan \theta = 3(1 + \tan^2 \theta) - 7$ $3 \tan^2 \theta - 4 \tan \theta - 4 = 0$ $(3 \tan \theta + 2)(\tan \theta - 2) = 0$ $\tan \theta = -\frac{2}{3}, 2$	M1 A1 m1 A1	$\sec^2 \theta = 1 + \tan^2 \theta$ allow $(3 \tan \theta - 2)(\tan \theta + 2)$ cao
	<u>Note</u> : No working shown m0 A0		
	$\theta = 63.4^\circ, 243.4^\circ$	B1	ft tan value, -1 each extra value in range
	$\theta = 146.3^\circ, 326.3^\circ$	B1	ft tan value if different sign. -1 each extra value in range

Note : Do not ft for other trig functions.

Q	Solution	Mark	Notes
10a(i)	Use of product rule	M1	$x^5 f(x) + g(x) \ln x$
	$x^5 \times \frac{1}{x} + 5x^4 \ln x$	A1	$f(x) = \frac{1}{x}$
		A1	$g(x) = 5x^4$ isw
10a(ii)	Use of quotient rule	M1	$\frac{(x^3 - 1)f(x) - e^{3x}g(x)}{(x^3 - 1)^2}$
	$\frac{(x^3 - 1)3e^{3x} - e^{3x}(3x^2)}{(x^3 - 1)^2}$	A1	$f(x) = 3e^{3x}$
		A1	$g(x) = 3x^2$ isw
10a(iii)	Use of chain rule	M1	$\frac{1}{2}(\tan x + 7x)^{-1/2} f(x)$
	$\frac{1}{2}(\tan x + 7x)^{-1/2}(\sec^2 x + 7)$	A1	$f(x) = (\sec^2 x + 7)$ isw
<u>Note</u> : $f(x), g(x) \neq 0$ or 1.			
10(b)	$3 \frac{dy}{dx} + 4y^2 + 8xy \frac{dy}{dx} - 15x^2 = 0$	B1	$3 \frac{dy}{dx} - 15x^2, 0$
		B1	$4y^2 + 8xy \frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{15x^2 - 4y^2}{3 + 8xy}$	B1	correct $\frac{dy}{dx}$ cao

Q	Solution	Mark	Notes
11(a)	$y = \frac{\sqrt{x^2 - 1}}{x}$ $x^2 y^2 = x^2 - 1$ $x^2(1 - y^2) = 1$ $x = \pm \frac{1}{\sqrt{1 - y^2}},$ $f^{-1}(x) = \frac{1}{\sqrt{1 - x^2}}, \text{ +ve since } x \geq 1$ Domain $[0, 1)$ 	M1	
		A1	
		A1	ft above for similar expression
		B1	
		G1	for $f(x)$ starting at $(1,0)$ with horizontal asymptote $y=1$ Or for $f^{-1}(x)$ starting at $(0,1)$ with vertical asymptote $x=1$ (does not need to be shown)
		G1	reflection in $y = x$, provided curve passes through $(1,0)$ or $(0,1)$
11(b)	$ff(x)$ cannot be formed because the range of $f(x)$ is not in the domain of $f(x)$.	E1	oe eg consideration of a single point.

Q	Solution	Mark	Notes
12(a)	Area of sector $OAB = \frac{1}{2} r^2 \theta$		
	Area triangle $OAB = \frac{1}{2} r^2 \sin \theta$	B1	either si
	Area of segment $= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta$	B1	si
	$3(\frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta) = \pi r^2$	M1	oe
	$\sin \theta = \theta - \frac{2\pi}{3}$	A1	convincing
12(b)(i)	$f(\theta) = \theta - \sin \theta - \frac{2\pi}{3}$		
	$f(2.6) = -0.00989647... < 0$	M1	
	$f(2.7) = 0.178225... > 0$		
	Change of sign, therefore $2.6 < \theta < 2.7$	A1	
12(b)(ii)	$f'(\theta) = 1 - \cos \theta$	B1	
	$\theta_{n+1} = \theta_n - \frac{\theta_n - \sin \theta_n - \frac{2\pi}{3}}{1 - \cos \theta_n}$	M1	si
	$\theta_0 = 2.6$		
	$\theta_1 = 2.6053296$	A1	1 st iteration correct si
			ft $f'(\theta) = 1 + \cos \theta$ only (2.6691...)
	$\theta_2 = 2.605325675$		
	$\theta = 2.605$ (correct to 3 d.p.)	A1	cao
<u>Note:</u> No marks for unsupported answer of 2.605.			
	$1 + \cos \theta$ in denominator, series is divergent.		

Q	Solution	Mark	Notes
13(a)	$\frac{dA}{dt} = kA$	B1	
13(b)	$\int \frac{dA}{A} = \int k dt$	M1	separate variables
	$\ln A = kt + (C)$	A1	
	$t = 0, A = 0.2$	m1	use of initial conditions
	$C = \ln 0.2$		
	$\ln \frac{A}{0.2} = kt$		
	$t = 1, A = 1.48$	m1	used
	$k = \ln(7.4) = 2.00148$		
	$e^k = 7.4$	A1	either k or e^k
	$(A =) 0.2e^{kt}$	A1	$k = 2, 2.00148, \ln(7.4)$
	$(A =) 0.2(7.4)^t$	A1	cao

Q	Solution	Mark	Notes
14(a)	$\frac{1}{2}e^{2x} - 2\cos 3x + C$	B1	one correct term
		B1	second correct term -1 if no +C.
14(b)	$(x^2 + \sin x)^7 + C$	B1	-1 if no +C(only once) .
14(c)	$I = \int x^{-2} \ln x \, dx = \left[\frac{x^{-1}}{-1} \ln x \right] - \int -x^{-1} \times \frac{1}{x} dx$	M1	$f(x)\ln x - \int f(x)\frac{1}{x} dx$
		A1	1 st term
		A1	2 nd term
	$I = -\frac{1}{x} \ln x + \int x^{-2} dx$		
	$I = -\frac{1}{x} \ln x - \frac{1}{x} + C$	A1	-1 if no +C (only once)
14(d)	$u = 2\cos x + 1; \quad du = -2\sin x \, dx$		
	$x = 0, u = 3; \quad x = \frac{\pi}{3}, u = 2$		
	$I = \int_3^2 -\frac{1}{2u^2} du = \frac{1}{2} \int_2^3 u^{-2} du$	M1	integrand au^{-2}
	$I = \frac{1}{2} \left[-\frac{1}{u} \right]_2^3$	A1	correct integration of u^{-2}
	$I = \frac{1}{2} \left[\frac{1}{2} - \frac{1}{3} \right]$	m1	correct use of correct limits
	$I = \frac{1}{12}$	A1	cao

Note: No marks for unsupported answer of 1/12.

Q	Solution	Mark	Notes
15	Assume that $\sqrt{6}$ is rational. Then there are (integers) a and b, with no common factor (except 1) such that $\sqrt{6} = \frac{a}{b}$	M1 m1	
OR	Assume $\sqrt{6} = \frac{a}{b}$, where a and b, are integers. a and b have no common factor (except 1).	(M1) (m1)	
THEN	Square both sides, $6 = \frac{a^2}{b^2}$ $6b^2 = a^2$ So (a^2 and thus)a is an even number, ($a = 2k$), $6b^2 = a^2 = (2k)^2 = 4k^2$ $3b^2 = 2k^2$ So (b^2 and thus)b is an even number. ($b = 2h$) So, a and b have a common factor 2. This is a contradiction. Hence $\sqrt{6}$ is irrational.	A1 A1	Dep on M1 Dep on M1
OR	$6b^2 = a^2$ So (a^2 and thus)a has a factor of 6, $a = 6k$ $6b^2 = a^2 = (6k)^2 = 36k^2$ $b^2 = 6k^2$ So (b^2 and thus)b has a factor of 6, $b = 6h$ So, a and b have a common factor 6. This is a contradiction. Hence $\sqrt{6}$ is irrational.	(A1) (A1)	Dep on M1 Dep on M1

Note: Also accept factor of 3.