GCE A LEVEL MARKING SCHEME

SUMMER 2019

A LEVEL (NEW)
MATHEMATICS
UNIT 4 APPLIED MATHEMATICS B
1300U40-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS

A2 UNIT 4 APPLIED MATHEMATICS B

SUMMER 2019 MARK SCHEME

SECTION A - STATISTICS

Qu. No.	Solution	Mark	Notes
1(a)	$P(W) = 0.2 \times 0.67 + 0.1 \times 0.55 + 0.7 \times 0.95$ $= 0.134 + 0.055 + 0.665$ $= 0.854 \left(\text{OR} \frac{427}{500}\right)$	M1 A1 A1	M1 for sight of at least four correct terms within addition formula
	OR $P(W) = 1 - P(F)$ $= 1 - (0.2 \times 0.33 + 0.1 \times 0.45 + 0.7 \times 0.05)$ $= 1 - (0.066 + 0.045 + 0.035)$ $= 1 - 0.146$ $= 0.854$	(M1) (A1) (A1)	$1 - P(F)$ needed for M1 and A1 Sight of at least four correct terms within addition formula for M1 SC1 for 0.146 $\left(\frac{73}{500}\right)$
(b)	$P(B\|W) = \frac{P(B \cap W)}{P(W)}$ $= \frac{0.1 \times 0.55}{0.854}$ $= 0.0644(0281 \ldots) \text{ OR } \frac{55}{854}$	M1 A1 **[5]**	FT their denominator from (a) provided $0 < P(W) < 1$ for possible M1A1 3sf required

Qu. No.	Solution	Mark	Notes
2(a)(i)	$\left(\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\right)\frac{125}{1296}$ or $0.096(4506)$	B1	3sf required
(ii)	$\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^5\times\frac{1}{6}=\frac{5}{36}+\frac{3125}{46656}$ $=0.13888\ldots+0.06697\ldots$ $=0.2059$ OR $\frac{9605}{46656}$	M2 A1	M1 for either product and addition. 3sf required
(iii)	$\left(\frac{5}{6}\right)^6\times\frac{1}{6}+\left(\frac{5}{6}\right)^7\times\frac{1}{6}+\left(\frac{5}{6}\right)^8\times\frac{1}{6}+\left(\frac{5}{6}\right)^9\times\frac{1}{6}$ $=\ 0.1734$	M2 A1	M1 for 2 or 3 correct products and addition. 3sf required
(b)	$P(\text{Alex wins})$ $=\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}+\left(\frac{5}{6}\right)^8\times\frac{1}{6}+\left(\frac{5}{6}\right)^{12}\times\frac{1}{6}+\cdots$ Attempting to sum an infinite geometric series $P(\text{Alex wins})=\frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^4}=\frac{216}{671}$ ag	M1 M1 A1 **[10]**	Allow M1 for stating that $a=\frac{1}{6}$ and $r=\left(\frac{5}{6}\right)^4=\frac{625}{1296}$. Both fractions needed for A1

Qu. No.	Solution	Mark	Notes
3 (a)	Each throw is independent AND The probability 0.2 stays the same for each throw. Valid reason. Eg. No because she may improve with practice. No, because she may get frustrated and the probability 0.2 might decrease. Yes. She is unable to improve her performance.	B1 B1	Ignore reference to statements based on fixed n and success/failure outcomes.
(b)	Two valid changes. As n increases: e.g. the distribution becomes more symmetrical e.g. more bell-shaped e.g mean increases e.g. variance/SD increases e.g. mode increases e.g. probability of mode decreases e.g. the probability of knocking at least one coconut off increases.	B1 B1 **[4]**	

Qu. No.	Solution	Mark	Notes
4(a)	Let the random variable X be the weight of kettlebells, $X\sim N(16{,}0.08^2)$ $P(X > 16.05) = 0.26599$	M1A1	M1 for any method. May be implied by correct answer. If answer incorrect, method must be shown. Use of tables gives 0.26435. 3sf required $z = \frac{16.05-16}{0.08}$ or $z = 0.625$ for M1
4(b)	Using limits of 15.95 and 16.05	B1	si
	$\bar{X} \sim N\left(16, \frac{0.08^2}{25}\right)$	B1	$P\left(Z > \frac{16.05-16}{\frac{0.08}{\sqrt{25}}}\right)$ oe for 15.95
	$P(\text{Reject}) = 1 - P\left(15.95 \le \bar{X} \le 16.05\right)$	M1	M1 for $z = 3.125$
	$= 0.00178$	A1	0.00174 given by tables.
(c)	$H_0: \mu = 16$ $H_1: \mu \neq 16$	B1	B0 for H_0: mean = 0. Population must be stated or implied.
	$P(\bar{X} > 16.02 \mid H_0) = 0.10565$	M1A1	M1 for $P\left(Z > \frac{16.02-16}{\frac{0.080}{\sqrt{25}}}\right)$ or $P(Z > 1.25)$
	$p - value = 2 \times 0.10565$ $= 0.2113$	A1	OR A1 for sight of 0.025
	$0.2113 > 0.05$, therefore insufficient evidence to reject H_0.	B1	OR 0.10656 > 0.025
	Sufficient evidence to suggest the new production method is adopted.	E1	E0 follows any contradictory statements Allow 'The new production method will be adopted'.
	OR		
	CV = 16.031(3592)	(M1A1)	
	TS = 16.02	(A1)	
	Since TS<CV, there is insufficient evidence to reject H_0.	(B1)	
	Sufficient evidence to suggest the new production method is adopted.	(E1)	
	OR		
	TS = $\frac{16.02-16}{0.080/\sqrt{25}} = 1.25$	(M1A1)	
	CV = 1.96	(A1)	
	Since TS<CV, there is insufficient evidence to reject H_0.	(B1)	
	Sufficient evidence to suggest the new production method is adopted.	(E1)	
		[12]	

Qu. No.	Solution	Mark	Notes
5 (a)	$H_0: \rho = 0$ $H_1: \rho < 0$	B1	B0 for H_0: correlation = 0. Population must be stated or implied.
	$n = 10$	B1	si
	$\text{TS} = -0.7617$	B1	B1 for use of or stating as test statistic
	CV = $(\pm)$ 0.7155	B1	
	Since $\lvert\text{TS}\rvert > 0.7155$ (or $\text{TS} < -0.7155$), there is sufficient evidence to reject H_0.	B1	
(b)	p-value $= 0.005215$ is less than 0.01	E1	Condone any significance level less than or equal to 0.1
	Bowling alley revenue is (positively) correlated with fish consumption per person.	E1	Do not allow weak correlation
	The correlations, despite being significant, seem irrelevant in both cases and therefore the board of directors should not implement any suggestions made by the manager based on his calculations.	E1	Award E1 for a comment that implies the board of directors should do nothing, or that the correlations are irrelevant with a reasonable course of action. Do not allow “the bowling alley should sell more fish and less margarine” Do not allow board of directors should do nothing because there is not correlation.
(c)	Valid comment. e.g. It is unclear what each data point represents. e.g. He has carried out the tests without considering how irrelevant the findings may be. e.g. It’s unclear whether this data is a sample, so no inferences can be made. e.g. It’s not clear what the population is. e.g. The data is not bivariate normal	E1	
		[9]	

SECTION B - DIFFERENTIAL EQUATIONS AND MECHANICS

Q6	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	M1	sin to cos or vice versa **and** coefficient multiplied $\mathbf{i}, \mathbf{j}$ retained
	$\mathbf{a} = -36\sin(3t)\,\mathbf{i} - 10\cos(2t)\,\mathbf{j}$	A1	cao
	$\mathbf{F} = (0\cdot5)(-36\sin(3t)\,\mathbf{i} - 10\cos(2t)\,\mathbf{j})$	A1	FT **a**
	$(\mathbf{F} = -18\sin(3t)\,\mathbf{i} - 5\cos(2t)\,\mathbf{j})$	**[3]**	
(b)	$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$	M1	sin to cos or vice versa **and** coefficient divided $\mathbf{i}, \mathbf{j}$ retained
	$\mathbf{r} = 4\sin(3t)\,\mathbf{i} + \frac{5}{2}\cos(2t)\,\mathbf{j}\ \ (+\mathbf{c})$	A1	cao
	$t = 0$, $\mathbf{r} = 4\mathbf{i} + 7\mathbf{j}$ $\mathbf{c} = 4\mathbf{i} + \frac{9}{2}\mathbf{j}$	m1	Used
	$\mathbf{r} = 4\sin(3t)\,\mathbf{i} + \frac{5}{2}\cos(2t)\,\mathbf{j}\ + 4\mathbf{i} + \frac{9}{2}\mathbf{j}$	A1	cao
	$\left(\mathbf{r} = 4(\sin(3t) + 1)\,\mathbf{i} + \frac{1}{2}(5\cos(2t) + 9)\,\mathbf{j}\right)$	**[4]**	
(c)	When $t = \frac{\pi}{2}$, $\mathbf{r} = 4\left(\sin\left(\frac{3\pi}{2}\right) + 1\right)\ \mathbf{i} + \frac{1}{2}\left(5\cos\left(\frac{2\pi}{2}\right) + 9\right)\ \mathbf{j}$ $\mathbf{r} = 2\,\mathbf{j}$	M1	Substitution, si, FT similar exp. for $\mathbf{r}$
	Distance of P from $O = 2$ (m)	A1	cao
		[2]	
	Total for Question 6	**9**	

Q7	Solution	Mark	Notes
(a)	Attempt to resolve in **two** directions	M1	No missing/extra forces,
	$X = \pm 21\cos\alpha \mp 8$ $= \pm 21 \times 0\cdot 8 \mp 8$ $= \pm 8\cdot 8$	A1	$21\cos\alpha$ and 8 opposing
	$Y = \pm 21\sin\alpha \mp 11$ $= \pm 21 \times 0\cdot 6 \mp 11$ $= \pm 1\cdot 6$	A1	$21\sin\alpha$ and 11 opposing
	$R = \sqrt{(\pm 1\cdot 6)^2 + (\pm 8\cdot 8)^2}$	m1	FT X, Y provided dim. correct
	$= 4\sqrt{5}$ or $8\cdot 9(4427\ldots)$ (N)	A1	cao
		[5]	
(b)	For any valid reason with correct working, e.g. • By Pythagoras, $21^2 > 11^2 + 8^2$. • Forces will form sides of a vector triangle when in equilibrium. However, $21 > 8 + 11$ so this is impossible. • Finding angles $21\cos\alpha = 8$ gives $\alpha = 67\cdot 6\ldots°$ $21\sin\alpha = 11$ gives $\alpha = 31\cdot 588\ldots°$ Values for α are **different**, so cannot be in equilibrium.	E1 **[1]**	
	Total for Question 7	**6**	

Q8	Solution	Mark	Notes
(a)	N2L applied to box $-0 \cdot 4v^2 = 2\frac{\mathrm{d}v}{\mathrm{d}t}$	M1	Dimensionally correct equation
	$5\frac{\mathrm{d}v}{\mathrm{d}t} + v^2 = 0$	A1	Convincing
		[2]	
(b)	$-\int \frac{5}{v^2}\mathrm{d}v = \int \mathrm{d}t$	M1	Separating variables **including** attempt to integrate
	$\frac{5}{v} = t \ (+C)$	A1	cao
	When $t = 0$, $v = 5$ $C = 1$	m1	Either limits or initial conditions used
	$v = \frac{5}{t+1}$	A1	cao
		[4]	
(c)	Takes an infinite amount of time to come to rest	E1	
		[1]	
	Total for Question 8	**7**	

Q9	Solution	Mark	Notes
(a)	Moments about base of **wire 2**	M1	Dim. correct, all forces/terms
	$mgd_C + T_1 \times 1 = mg(1 + d_A) + mg(1 - d_B)$	A1	cao
	$T_1 = mg + mgd_A + mg - mgd_B - mgd_C$		
	$T_1 = mg(2 + d_A - d_B - d_C)$	A1	Convincing
	Resolve vertically $T_1 + T_2 = 3mg$	M1 A1	Dim. correct, all forces/terms
	$T_2 = 3mg - T_1$		
	$T_2 = mg(1 - d_A + d_B + d_C)$ oe	A1	cao
		[6]	
(b)	(i) For maximum tension in T_1		
	$d_A = 0 \cdot 3$ (maximum) $d_B = 0 \cdot 1$ (minimum) $d_C = 0$ (minimum)	M1	Used with at least 2 out of 3 correct
	$T_1 = mg(2 + 0 \cdot 3 - 0 \cdot 1 - 0)$		
	Maximum $T_1 = 2 \cdot 2mg$	A1	cao
	(ii) Maximum $T_2 = 2 \cdot 2mg$, due to symmetry	B1	FT candidate's value of T_1. Reason **MUST** be given
		[3]	
	Total for Question 9	**9**	

Q10	Solution	Mark	Notes
(a)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with $\mathbf{u} = (30\mathbf{i} - 1\cdot 4\mathbf{j}), \mathbf{a} = -g\mathbf{j}, t = \frac{4}{7}$	M1	Used, Allow $\mathbf{a} = \pm g\mathbf{j}$
	$\mathbf{v} = (30\mathbf{i} - 1\cdot 4\mathbf{j}) + (-g\mathbf{j})\left(\frac{4}{7}\right)$		
	$\mathbf{v} = (30\mathbf{i} - 7\mathbf{j})$	A1	cao
	$\lvert\mathbf{v}\rvert = \sqrt{(30)^2 + (-7)^2} = \sqrt{949}$		
	$\lvert\mathbf{v}\rvert = 30\cdot 8 \quad (1\text{ dp}) \quad (\text{ms}^{-1})$	A1	FT $\mathbf{v} = (30\mathbf{i} + k\mathbf{j})$
		[3]	
	<u>Alternative solution(s)</u>		
	Working vertically using $v = u + at$, with $u = \pm 1\cdot 4, a = \pm 9\cdot 8, t = \frac{4}{7}$	(M1)	
	$v = \pm 1\cdot 4 + (\pm 9\cdot 8)\left(\frac{4}{7}\right) = \pm 7$	(A1)	$1\cdot 4$ **and** $9\cdot 8$ same sign giving $v = \pm 7$ or $\pm 7\mathbf{j}$
	or		
	Working vertically using $v^2 = u^2 + 2as$, with $u = \pm 1\cdot 4, a = \pm 9\cdot 8, s = \pm 2\cdot 4$	(M1)	
	$v^2 = (\pm 1\cdot 4)^2 + 2(\pm 9\cdot 8)(\pm 2\cdot 4)$		
	$v = \pm 7$	(A1)	$1\cdot 4$ **and** $9\cdot 8$ **and** $2\cdot 4$ same sign giving $v = \pm 7$ or $\pm 7\mathbf{j}$
	followed by speed $= \sqrt{(30)^2 + (\pm 7)^2} = \sqrt{949}$		
	$= 30\cdot 8 \ (1\text{ dp}) \quad (\text{ms}^{-1})$	(A1)	FT v provided 30 is used
		([3])	

(b)	i) $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$, with $\mathbf{u} = (30\mathbf{i} - 1\cdot 4\mathbf{j})$, $\mathbf{a} = -g\mathbf{j}, t = 0\cdot 4$	M1	Used, Allow $\mathbf{a} = \pm g\mathbf{j}$
	$\mathbf{s} = (30\mathbf{i} - 1\cdot 4\mathbf{j})(0\cdot 4) + \frac{1}{2}(-g\mathbf{j})(0\cdot 4)^2 + (2\cdot 4\mathbf{j})$ $\mathbf{s} = 12\mathbf{i} - 1\cdot 344\mathbf{j} + 2\cdot 4\mathbf{j}$	m1	Adding initial position vector of $(2\cdot 4\mathbf{j})$
	$\mathbf{s} = 12\mathbf{i} + 1\cdot 056\mathbf{j}$	A1	cao
	<u>Alternative solution for i)</u>		
	Working vertically using $s = \mathrm{u}t + \frac{1}{2}\mathrm{a}t^2$, with $u = \pm 1\cdot 4, a = \pm 9\cdot 8, t = 0\cdot 4$	(M1)	
	$s = (\pm 1\cdot 4)(0\cdot 4) + \frac{1}{2}(\pm 9\cdot 8)(0\cdot 4)^2$		
	$s = \pm 1\cdot 344$		Allow $\pm 1\cdot 344\mathbf{j}$
	Dist. above ground $= 2\cdot 4 - 1\cdot 344 (= 1\cdot 056)$	(m1)	FT s above, i.e. $2\cdot 4 - \lvert s\rvert$
	Horizontally, $x = 30 \times 0\cdot 4 = 12$		
	$\mathbf{s} = 12\mathbf{i} + 1\cdot 056\mathbf{j}$ (m)	(A1)	cao
	ii) Comparison of **i, j** coefficients yields $x = 12$ **and** Net clearance $= 1\cdot 056 - 0\cdot 9 = 0\cdot 156$ (m) ≈ 16 cm	A1 **[4]**	Value of x MUST be stated Convincing
(c)	i) Valid Reason, e.g. • Dimensions of the ball not considered • Air resistance/wind (friction) • Spin/rotation of the ball	E1	
	ii) Sensible improvement to account for any of the above	E1 **[2]**	
	Total for Question 10	**9**	