wjec cbac

GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
MATHEMATICS
UNIT 1 PURE MATHEMATICS A
2300U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

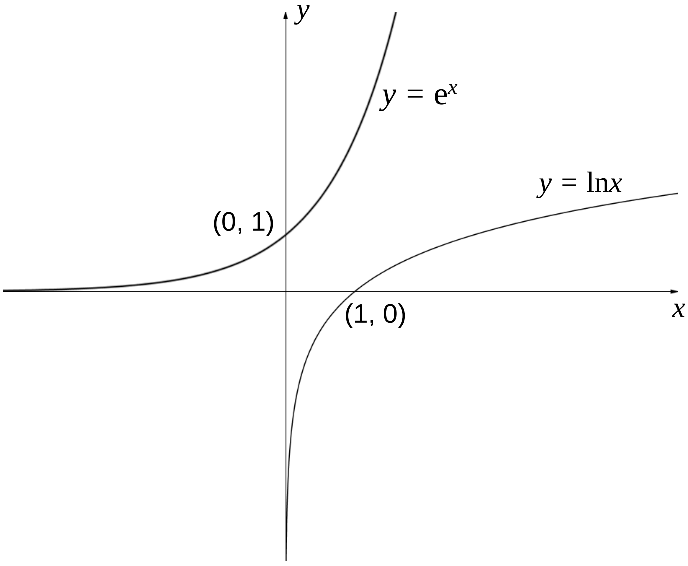
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS MATHEMATICS

UNIT 1 PURE MATHEMATICS A

SUMMER 2022 MARK SCHEME

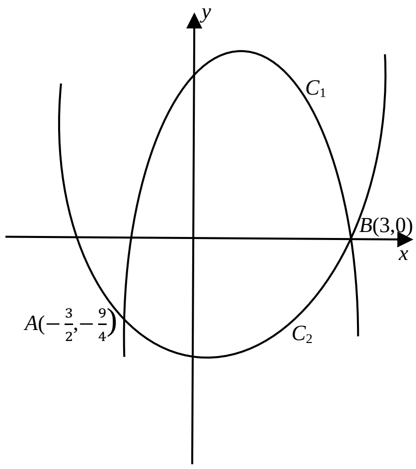
Q	Solution	Mark	Notes
1	$y = \ln x$	B1	Allow $y = \log_e x$ May be seen on graph
			
		B1	graph of $y = e^x$ and (0,1)
		B1	graph of $y = \ln x$ and (1,0)
		<u>If B0 B0</u>	
		SC1	both graphs correctly drawn, but intercepts missing or incorrect
	OR		
		SC1	correct intercepts but incorrect graphs

Q	Solution	Mark	Notes
2	$5\sqrt{48} = 20\sqrt{3}$	B1	
	$(2\sqrt{3})^3 = 24\sqrt{3}$	B1	
	$\frac{2+5\sqrt{3}}{5+3\sqrt{3}} = \frac{(2+5\sqrt{3})(5-3\sqrt{3})}{(5+3\sqrt{3})(5-3\sqrt{3})}$	M1	multiplying by conjugate M0 if multiplying by conjugate not shown
	$= -\frac{1}{2}(10 - 6\sqrt{3} + 25\sqrt{3} - 45)$	A1	for numerator
		A1	for denominator (25 – 27)
	$= -\frac{1}{2}(19\sqrt{3} - 35)$		
	Expression = $\frac{1}{2}(35 - 27\sqrt{3})$	A1	cao, any correct simplified form

Q	Solution	Mark	Notes
3(a)	Grad. of $L_1 = \frac{increase\ in\ y}{increase\ in\ x}$	M1	
	Grad. of $L_1 = \frac{-1-5}{3-0} = -2$	A1	
	Equ of L_1 is $y - 5 = -2x$	A1	any correct form Mark final answer
	$y + 2x = 5$		
3(b)	$y = \frac{1}{2}x$	B1	ft grad L_1 any correct form Mark final answer
3(c)	At C, $\frac{1}{2}x + 2x = 5$	M1	oe
	$x = 2$, $y = 1$	A1	ft their (a) and (b)
	C is the point (2, 1)		
	Area $OAC = \frac{1}{2} \times OA \times$ (x-coord of C)	M1	
	Area $OAC = (\frac{1}{2} \times 5 \times 2) = 5$	A1	ft their 'x-coord of C'
	OR		
	Area $OAC = \frac{1}{2} \times OC \times AC$	(M1)	
	$OC = \sqrt{2^2 + 1^2} = \sqrt{5}$		
	$AC = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$		
	Area $OAC = (\frac{1}{2} \times \sqrt{5} \times 2\sqrt{5}) = 5$	(A1)	ft their coordinates of C

Q	Solution	Mark	Notes
3(d)	Gradient of L_3 = -2	M1	
	Either		
	Equ of L_3 is y -2 = -2(x -4)	A1	ft their gradient of L_1 any correct form ISW
	OR		
	Equ of L_3 is y = -2x + c		
	2 = -2 × 4 + c		
	c = 10		
	Equ of L_3 is y = -2x +10	(A1)	ft their gradient of L_1
3(e)	Using similar triangles,		
	Area ODE = $2^2 \times 5 = 20$	B1	ft their (c)
	OR		
	Area = $\frac{1}{2} \times OE \times$ (x-coord of D)		
	Area = $\frac{1}{2} \times 10 \times 4 = 20$	(B1)	

Q	Solution	Mark	Notes
4	$x^2 + 3x - 6 > 4x - 4$		
	$x^2 - x - 2$ (> 0)	M1	oe Allow 1 slip terms all collected on one side
	$(x + 1)(x - 2)$ (> 0)	A1	si condone ‘=’ ft their quadratic
	Critical values, -1 and 2	A1	si cao
	$x < -1$ or $x > 2$	A1	ft their critical values condone ‘,’, or nothing A0 for ‘and’ Mark final answer

	Solution	Mark	Notes
5(a)	$-x^2 + 2x + 3 = x^2 - x - 6$	M1	
	$2x^2 - 3x - 9 = 0$	A1	
	$(2x + 3)(x - 3) = 0$		
	$x = -\frac{3}{2}, 3$	A1	or one correct pair A0 A0 if no workings seen
	$y = -\frac{9}{4}, 0$	A1	all correct
	$A(-\frac{3}{2}, -\frac{9}{4})$ $B(3,0)$		or other way round
			If 0 marks, award SC1 for sight of (3,0)
5(b)			
		M1	at least one quadratic curve
		A1	one cup, one hill
		A1	graphs all correct with correct points of intersection FT points of intersection where possible

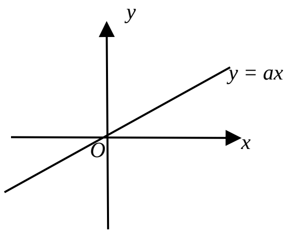
5(c)	Area enclosed by curves to the right of the *y*-axis		ft for equivalent diagram
		B1	for 1 correct region
		B1	for 2^{nd} correct region -1 for each additional incorrect region

Q	Solution	Mark	Notes
6(a)	Statement B is false		
	<u>Two negative numbers</u>:		
	Correct choice of numbers, eg		
	$x = -25$, $y = -4$,	M1	
	Correct verification, eg		
	$x + y = -29$		
	$2\sqrt{xy} = 2 \times \sqrt{(-25) \times (-4)}$	A1	both substitutions
	$2\sqrt{xy} = 20$		
	Since $-29 < 20$ statement B is false.	A1	oe
	<u>One positive number, one negative number</u>:		
	Correct choice of numbers, eg		
	$x = 1$, $y = -4$,	(M1)	
	Correct verification, eg		
	$x + y = -3$		
	$2\sqrt{xy} = 2 \times \sqrt{(1) \times (-4)}$	(A1)	both substitutions
	$2\sqrt{xy} = 2\sqrt{-4}$		
	$2\sqrt{-4}$ is not real, statement B is false.	(A1)	oe

Q	Solution	Mark	Notes
6(b)	Statement A is true		
	Either		
	$x^2+y^2 \geq 2xy$		
	$x^2 - 2xy + y^2 \geq 0$	M1	
	$(x - y)^2 \geq 0$, which is always true	A1	
	Therefore, Statement A is true		
OR			
	Consider $(x - y)^2 \geq 0$	(M1)	
	$x^2 - 2xy + y^2 \geq 0$		
	$x^2+y^2 \geq 2xy$	(A1)	

Q	Solution	Mark	Notes
7(a)	$A(2, 3)$	B1	
	A correct method for finding the radius,		
	e.g., $(x - 2)^2 + (y - 3)^2 = 4^2$	M1	
	Radius = 4	A1	
7(b)	At points of intersection		
	$x^2 + (x + 5)^2 - 4x - 6(x + 5) - 3 = 0$	M1	
	$2x^2 - 8 = 0$	A1	oe or $2y^2 - 20y + 42 = 0$ All terms collected
	$x = -2, 2$	A1	or $y = 3, 7$ or 1 correct pair
	$y = 3, 7$	A1	or $x = -2, 2$ all correct
	$P(-2, 3)$ $Q(2,7)$		or $P(2,7)$, $Q(-2, 3)$
7(c)	Attempt to find, B, the midpoint of PQ	M1	ft their P and Q
	$B(0, 5)$		
	$PB = \sqrt{(-2-0)^2 + (3-5)^2} = \sqrt{8} = 2\sqrt{2}$	A1	ft their P and Q
	OR		
	$PB = \frac{1}{2} PQ = \frac{1}{2}\sqrt{(-2-2)^2 + (3-7)^2}$	(M1)	
	$PB = \frac{1}{2} 4\sqrt{2}$		
	$PB = 2\sqrt{2}$	(A1)	ft their P and Q

Q	Solution	Mark	Notes
7(d)	Area = quarter circle – triangle *APQ*	M1	
	Area = $\frac{1}{4} \times \pi \times 4^2 - \frac{1}{2} \times 4 \times 4$	A1	
	Area = $4\pi - 8$		answer given

Q	Solution	Mark	Notes
8(a)		B1	Straight line through the origin, positive or negative gradient
8(b)	Mary's pay = $120 \times \frac{2}{3}$	M1	Divide by 3
		oe e.g. $3m = 120$	
		M1	oe × by 2
	Mary's pay = £80	A1 Unsupported answer of £80 award M1A1A1	
8(c)	$P = 1013 \times 0.88^{\frac{H}{1000}}$	B1	
	When $H = 8848$, $P = 1013 \times 0.88^{\frac{8848}{1000}}$	M1	e.g. $P = 1013 \times 0.88^{H}$ Allow $P = 1013 \times 0.988^{H}$
	$P = 326.8828$ or 327 (units)	A1	Allow answers in the range 324 to 330

Q	Solution	Mark	Notes
9	Discriminant = $(2k)^2 – 4 \times 1 \times 8k$	B1	An expression for $b^2 – 4ac$
	Discriminant = $4k^2 − 32k$		
	If no real roots, discriminant < 0	M1	May be implied by later work M0 if discriminant given in terms of k **and** x
	$k(k – 8) < 0$		
	Critical values, $k = 0, 8$	B1	si ft their quadratic discriminant if B0 awarded previously
	$0< k < 8$	A1	ft their 2 critical values provided M1 awarded

Q	Solution	Mark	Notes
10	$\ln 2^x = \ln 53$	M1	taking ln or log to any base of both sides.
	$x\ln 2 = \ln 53$	A1	use of power law
	$x = \frac{\ln 53}{\ln 2}$		
	$x = 5.727920455$		
	$x = 5.73$	A1	cao Must be to 2dp

Note:

- No workings M0
- $x = \log_2 53$, award M1A1

Q	Solution	Mark	Notes
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 10 + 6x - 3x^2$	M1	At least one correct term
	Attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$	m1	
	Grad of tangent at $C = 10$	A1	cao
	Equation of tangent at C is		
	$y - 24 = 10(x - 2)$	m1	oe
	$y = 10x + 4$		
	D is the point (0, 4)	A1	cao
11(b)	Area of trapezium = $\frac{1}{2}(4 + 24) \times 2$ (= 28)	B1	ft their $D(0,k)$, $0 < k < 24$
	A under curve = $\int_0^2 (10x + 3x^2 - x^3)\mathrm{d}x$	M1	attempt to integrate, at least one term correct, limits not required
	$= \left[5x^2 + x^3 - \frac{x^4}{4}\right]_0^2$	A1	correct integration, limits not required
	= (20 + 8 – 4) – (0)	m1	use of limits
	(= 24)		
	Shaded area = area (trap – under curve)	m1	
	Shaded area = 4	A1	cao

Note: Must be supported by workings

Q	Solution	Mark	Notes
11(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 10 + 6x - 3x^2$		FT their $\frac{\mathrm{d}y}{\mathrm{d}x}$ where possible
	At stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1	
	$10 + 6x - 3x^2 = 0$		
	$3x^2 - 6x - 10 = 0$		
	$x = \frac{6 \pm \sqrt{(-6)^2 - 4\times 3\times(-10)}}{6}$	m1	attempt to solve quadratic
	$x = -1.08,\ 3.08$ or $\frac{3\pm\sqrt{39}}{3}$	A1	any correct form
	Required range is $-1.08 < x < 3.08$	A1	

<u>Alternative Solution</u>

Q	Solution	Mark	Notes
11(c)	$f'(x) = 10 + 6x - 3x^2$		FT their $f'(x)$ where possible
	For increasing function, $f'(x) > 0$	(M1)	
	$10 + 6x - 3x^2 > 0$		
	$3x^2 - 6x - 10 < 0$		
	$x = \frac{6 \pm \sqrt{(-6)^2 - 4\times 3\times(-10)}}{6}$	(m1)	attempt to solve quadratic
	$x = -1.08,\ 3.08$ or $\frac{3\pm\sqrt{39}}{3}$	(A1)	any correct form
	Required range is $-1.08 < x < 3.08$	(A1)	

Q	Solution	Mark	Notes
12(a)	$f(x) = 2x^3 - x^2 - 5x - 2$		
	$f(-1) = -2 - 1 + 5 - 2 = 0$	M1	one use of factor theorem
	$(x + 1)$ is a factor	A1	oe
	$f(x) = (x + 1)(2x^2 + px + q)$	M1	at least one of p, q correct
	$f(x) = (x + 1)(2x^2 - 3x - 2)$	A1	oe (see note below*) cao
	$f(x) = (x + 1)(2x + 1)(x - 2)$	m1	coeffs of x^2 multiply to give 2 constant terms multiply to their q or formula with correct a,b,c
	$x = -1, -\frac{1}{2}, 2$	A1	cao

<u>Note</u>:

- Answers only with no workings 0 marks
- * $f(x) = (x - 2)(2x^2 + 3x + 1)$
- * $f(x) = (2x + 1)(x^2 - x - 2)$

12(b)	$\cos(2\theta - 51^o) = 0.891$		
	$2\theta - 51^o = 27^o, (-27^o)$	B1	
	$\theta = 39^o$	B1	
	$\theta = 12^o$	B1	
			-1 each extra root up to 2
			Ignore roots outside $0° < \theta < 180°$

Q	Solution	Mark	Notes	
13	Required term = $\binom{5}{3}(2)^{5-3}(-3)^3$	B1	$\binom{5}{3}$	oe
		B1	$(2)^{5-3}$	oe
		B1	$(-3)^3$	oe
	Required term = $10 \times 4 \times (-27)$			
	Required term = -1080	B1	ISW	

Q	Solution	Mark	Notes
14(a)	Attempt to differentiate	M1	
	$f'(x) = 9x^2 - 10x + 1$	A1	
	$9x^2 - 10x + 1 = 0$	m1	
	$(9x - 1)(x - 1) = 0$		
	$x = \frac{1}{9}, y = -\frac{1445}{243} = -5.9465$	A1	or $x = \frac{1}{9}, 1$
	$x = 1, y = -7$	A1	all correct
	$f''(x) = 18x - 10$	M1	oe ft quadratic $f'(x)$
	$x = \frac{1}{9}$,($f(x) = -5.9465$) is a maximum	A1	ft their x value
	$x = 1$, ($f(x) = -7$) is a minimum	A1	ft their x value provided different conclusion

Note: if $f''(x)$ is incorrectly found from their $f'(x)$, maximum marks M1A1A0

Q	Solution	Mark	Notes
14(b)(i)	Rewriting the equation		
	To give $f(x) = 3x^3 - 5x^2 + x - 6$ on one side.	M1	oe
	$3x^3 - 5x^2 + x - 6 = -7$,		
	2 (distinct roots)	A1	
14(b)(ii)	To give $f(x) = 3x^3 - 5x^2 + x - 6$ on one side	M1	oe
	$3x^3 - 5x^2 + x - 6 = -6.5$		
	3 (distinct roots)	A1	

Note: 14b – 0 marks for unsupported answers

Q	Solution	Mark	Notes
15	$\frac{(x^2y)^3}{x^2y^2} \times \frac{9}{x^2y^2} = 36$	B1	one use of subtraction law
		B1	one use of addition law
		B1	one use of power law
	$4y = x^2$	B1	oe for a correct equation after the removal of logs
	$\log_a\left(\frac{y}{x+3}\right) = \log_a 1$	(B1)	for use of the subtraction law if not previously awarded.
	$y = x + 3$	B1	or $x = y - 3$
	$4y = 4x + 12 = x^2$	M1	or $4y = (y - 3)^2$
	$x^2 - 4x - 12 = 0$		or $y^2 - 10y + 9 = 0$
	$(x + 2)(x - 6) = 0$		or $(y - 1)(y - 9) = 0$
	$x = -2, 6$	A1	cao or $y = 1, 9$ or 1 correct pair
	$y = 1, 9$	A1	cao or $x = -2, 6$ all correct
	$x = -2$ and $y = 1$, $x = 6$ and $y = 9$		
OR			
	$3(2\log_a x + \log_a y) - 2(\log_a x + \log_a y)$	(B1B1B1)	one for each use of laws
	$+ \log_a 9 - 2(\log_a x + \log_a y) = \log_a 36$	(B1)	correct equation
	$2\log_a x - \log_a y = \log_a 4$		
	$\log_a y - \log_a(x + 3) = 0$		
	$2\log_a x - \log_a(x + 3) = \log_a 4$	(M1)	solve simultaneously
	$x^2 - 4x - 12 = 0$	(A1)	
	$(x + 2)(x - 6) = 0$		
	$x = -2, 6$	(A1)	
	$y = 1, 9$	(A1)	
	$x = -2$ and $y = 1$, $x = 6$ and $y = 9$		

Q	Solution	Mark	Notes
16(a)	$\|\mathbf{a}\| = \sqrt{2^2 + 1^2}$	M1	correct method
	$\|\mathbf{a}\| = \sqrt{5}$		
	Required unit vector = $\frac{1}{\sqrt{5}}(2\mathbf{i} - \mathbf{j})$	A1	
16(b)	$\theta = \tan^{-1}(\pm 3)$	M1	
	$\theta = (\pm)71.6°\ (288.4°)$	A1	Accept 72° or 288°
16(c)(i)	$\mu\mathbf{a} + \mathbf{b} = \mu(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} - 3\mathbf{j})$	B1	Mark final answer
	$\mu\mathbf{a} + \mathbf{b} = (2\mu + 1)\mathbf{i} - (\mu + 3)\mathbf{j}$		
16(c)(ii)	If parallel to $4\mathbf{i} - 5\mathbf{j}$,		
	$(2\mu + 1)\mathbf{i} - (\mu + 3)\mathbf{j} = k(4\mathbf{i} - 5\mathbf{j})$	M1	or $k((2\mu + 1)\mathbf{i} - (\mu + 3)\mathbf{j}) = (4\mathbf{i} - 5\mathbf{j})$ Both sides in terms of **i** and **j**
	$2\mu + 1 = 4k$ and $\mu + 3 = 5k$	A1	ft (c)(i)
	Solving simultaneously	m1	any correct method
	$(k = \frac{5}{6})$		
	$\mu = \frac{7}{6}$	A1	cao
	<u>Alternative solution</u>		
	If parallel to $4\mathbf{i} - 5\mathbf{j}$,		
	$\frac{2\mu+1}{\mu+3} = \frac{4}{5}$	(M1A1)	ft (c)(i)
	$10\mu + 5 = 4\mu + 12$	(m1)	
	$6\mu = 7$		
	$\mu = \frac{7}{6}$	(A1)	cao