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**WELSH JOINT EDUCATION COMMITTEE**  
**CYD-BWYLLGOR ADDYSG CYMRU**

**General Certificate of Education**  
**Advanced Subsidiary/Advanced**

**Tystysgrif Addysg Gyffredinol**  
**Uwch Gyfrannol/Uwch**

**MARKING SCHEMES**

**JANUARY 2006**

**MATHEMATICS**  
**(New Specification)**

**WJEC**  
**CBAC**

## **INTRODUCTION**

The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

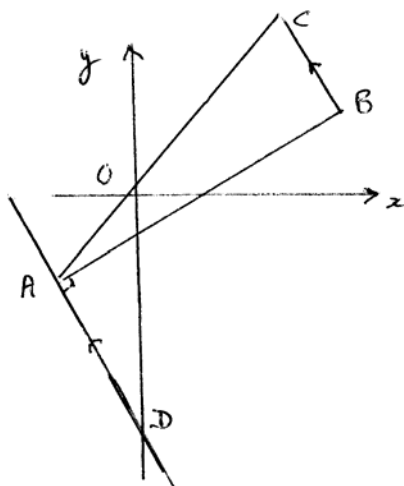
# MATHEMATICS C1

1. (a) Gradient of  $AB = \frac{1+3}{6+2} = \frac{1}{2}$

M1 (correct method)

A1 (grad =  $\frac{1}{2}$ )

(b)



Gradient of  $BC = \frac{3-1}{k-6} = \frac{2}{k-6}$

M1 (use of  $m_1 m_2 = -1$ ,  
seen or implied)

Then  $\frac{1}{2} \times \frac{2}{k-6} = -1$

M1 (method of finding  
equation in  $k$ )

$$\begin{aligned} k-6 &= -1 \\ k &= 5 \end{aligned}$$

A1 (C.A.O.) (convincing)

(c) Gradient of  $L = -2$

B1 (F.T. for same gradient as  
 $BC$  or  $-\frac{1}{\text{grad} AB}$ )

Equation of  $L$  is  $y+3 = -2(x+2)$

B1 (give mark here)

(d) Coords of  $D$  :  $x=0, y=-7$

B1(F.T. unsimplified equation of  $L$ )

$$CD = \sqrt{(5-0)^2 + (3+7)^2} = \sqrt{125}$$

M1 (correct formula)

A1 ( $\sqrt{\text{single no}}$ , F.T. derived  
coords of  $D$ )

2. (a)  $4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3}$

$= 5\sqrt{3}$

B1, B1, B1

B1 (F.T. one slip, answer of form  $k\sqrt{3}$ )

(b) 
$$\frac{(2 + \sqrt{7})(3 - \sqrt{7})}{(3 + \sqrt{7})(3 - \sqrt{7})} = \frac{6 - 2\sqrt{7} + \sqrt{7} - 7}{9 - 7}$$

M1 (correct rationalising)

A1 numerator with  $(\sqrt{7})^2 = 7$ ,  
allow  $2 \times 3$

A1 (denominator with no surds)

$$= \frac{\sqrt{7} - 1}{2}$$

A1 (F.T. one slip)

8

3.  $\frac{dy}{dx} = 8x - 7$

$= 9$  at  $(2, 4)$

B1 (correct differentiation)

B1 (numerical result, F.T. one slip)

Gradient of normal  $= -\frac{1}{9}$

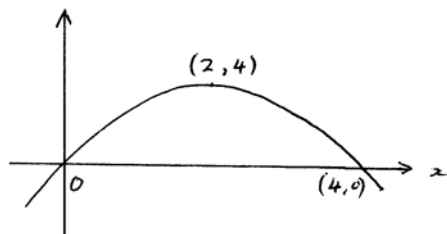
M1  $\left( \frac{-1}{\text{gradient of tgt}} \right)$

Equation is  $y - 4 = -\frac{1}{9}(x - 2)$

A1 (F.T. candidate's gradient of tangent)

4

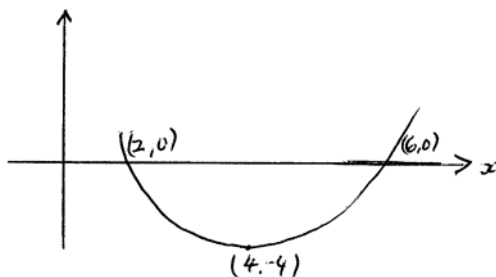
4. (a)



M1 (reflection in  $x$  - axis)

A1 (correct points)

(b)



M1 ( $x$  translation to right)

B1 (st.pt) (on diagram or stated)

A1 (points of intersection)

5

|                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>5. For no real roots,</p> $4^2 - 4(k+2)(k+5) < 0$ $4 - k^2 - 7k - 10 < 0$ $k^2 + 7k + 6 > 0$ $(k+6)(k+1) > 0$ $k < -6 \text{ (and/or) } k > -1$ <p>or</p> $k < -6, k > -1$ $-1 < k < -6 \text{ or } k > -1 \text{ or } k < -6$                                                                                                        | <p>M1 (<math>b^2 - 4ac</math>, correct <math>b</math>, <math>a</math> or <math>c</math> correct)</p> <p>A1 (correct)</p> <p>M1 (<math>b^2 - 4ac &lt; 0</math>)</p> <p>A1 (convincing)</p> <p>B1 (fixed pts, <math>-1, -6</math>)</p> <p>B2 (F.T. fixed points)</p> <p>All gain B1</p> <p>7</p>                                                                                 |
| <p>6. (a) <math>f(2) = 4</math></p> $8a - 4 - 14 + 6 = 4$ $8a = 16$ $a = 2$ <p>(b) <math>f(1) = 2 - 1 - 7 + 6 = 0</math></p> <p><math>x - 1</math> is a factor</p> $2x^3 - x^2 - 7x + 6 = (x - 1)(2x^2 + x - 6)$ $= (x - 1)(2x - 3)(x + 2)$ <p>Roots are <math>1, \frac{3}{2}, -2</math></p>                                             | <p>M1 (remainder theorem or division with remainder equated to 4)</p> <p>A1</p> <p>M1 (correct use of factor theorem with appropriate factor mentioned)</p> <p>A1 (correct factor)</p> <p>m1 (<math>2x^2 + ax + b</math>, <math>a</math> or <math>b</math> correct, any method)</p> <p>A1 (correct quad. factor)</p> <p>A1 (3 factors and 3 roots, F.T. one slip)</p> <p>7</p> |
| <p>7. (a) <math>(3x + 2)^3 = (3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + 2^3</math></p> $= 27x^3 + 54x^2 + 36x + 8$ <p>(b) <math>\frac{n(n-1)}{2}(2)^2 = 2(2n)</math></p> $n = 3$                                                                                                                                                                 | <p>B3 (<math>-1</math> for each error, any method)</p> <p>M1 (<math>{}^nC_2 2^p = 2k{}^nC_1, k=2, \frac{1}{2}, p=1,2</math>)</p> <p>A2 (<math>\frac{n(n-1)}{2} 2^2 = 2(2n)</math>)</p> <p>A1 (C.A.O.)</p> <p>7</p>                                                                                                                                                             |
| <p>8. (a) Let <math>y = 2x^2 - 5x + 3</math></p> $y + \Delta y = 2(x + \Delta x)^2 - 5(x + \Delta x) + 3$ $\Delta y = 4x\Delta x + 2(\Delta x)^2 - 5$ $\frac{\Delta y}{\Delta x} = 4x + 2\Delta x - 5$ <p>Let <math>\Delta x \rightarrow 0</math></p> $\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = 4x - 5$ | <p>B1</p> <p>M1 (attempt to find <math>\Delta y</math>)</p> <p>A1</p> <p>M1 (divide by <math>\Delta x</math> and let <math>\Delta x \rightarrow 0</math>)</p> <p>A1 (award only if some mention of limit, clear presentation and no abuse of notation)</p>                                                                                                                     |

(b)  $\frac{dy}{dx} = -\frac{a}{x^2} + 3x^{\frac{1}{2}}$  (o.e.) B1, B1

$$-\frac{a}{4^2} + 3 \times 4^{\frac{1}{2}} = 7$$

$a = -16$  M1 ( $f'(4)=7$ , reasonable diffn)  
A1 (C.A.O.)

9

9. (a)  $23 + 6x - x^2 = 32 - (x - 3)^2$  B1 ( $-(x - 3)^2$ )  
B1 (32)

Greatest value = 32  
when  $x = 3$  B1 (F.T. candidate's  $b$  and  $a$ )  
B1

(b)  $\frac{1}{30 + 6x - x^2} = \frac{1}{7 + 23 + 6x - x^2}$  A1

Least value =  $\frac{1}{39}$  A1 (F.T.  $b, a$ )

6

10. (a)  $\left(\frac{dy}{dx} = 0\right)$  B1  $\left(\frac{dy}{dx}\right)$

$$12x - 6x^2 = 0$$

$x = 0, 2$  M1  $\left(\frac{dy}{dx} = 0\right)$

When  $x = 0, y = 2$ ; when  $x = 2, y = 10$  A1 (either root, F.T. one slip)

$$\frac{d^2y}{dx^2} = 12 - 12x$$

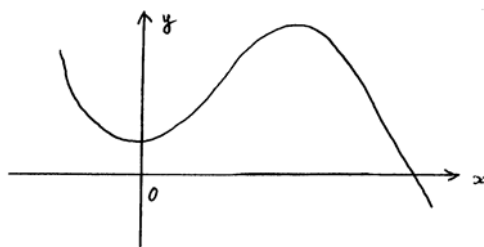
$x = 0, \frac{d^2y}{dx^2} = 12 > 0$  min. A1 (both C.A.O.)

$x = 2, \frac{d^2y}{dx^2} = -12 < 0$  max. M1 (any method)

A1 (F.T.  $x$  values)

A1

(b)



B1 (shape, negative cubic)

B1, B1 (derived stat pts)

(c) One real root as graph crosses  $x$ -axis once M1 (reason)  
A1 (F.T. graph)

12

## MATHEMATICS C2

1. (a)  $h = 0.2$  M1 (correct formula  $h = 0.2$ )
- Integral  $\approx \frac{0.2}{2} [0.5 + 0.3333333 + 2(0.4980080 + 0.48449612$  B1 (4 values)  
 $+ 0.4512635 + 0.3980892)]$  B1 (2 values)  
 $\approx 0.4497$  A1 (F.T. one slip)
- S. Case  $h = \frac{1}{6}$  M1 (correct formula,  $h = \frac{1}{6}$ )
- Integral  $\approx \frac{1}{12} [0.5 + 0.3333333 + 2(0.4988453 + 0.4909091$  B1 (all values)  
 $+ 0.4705882 + 0.4354838 + 0.3877917)]$
- $\approx 0.4500$  A1 (F.T. one slip)
2. (a)  $4 \cos^2 \theta - \cos \theta = 2(1 - \cos^2 \theta)$  M1 (correct use of  $\sin^2 \theta + \cos^2 \theta = 1$ )
- $6 \cos^2 \theta - \cos \theta - 2 = 0$  M1 (attempt to solve quadratic in  $\cos \theta$ , correct formula)
- $(3 \cos \theta - 2)(2 \cos \theta + 1) = 0$
- $\cos \theta = \frac{2}{3}, -\frac{1}{2}$  A1 (C.A.O.)
- $\theta = 48.2^\circ, 311.8^\circ, 120^\circ, 240^\circ$  B1 (48.2, 311.8°)  
B1 (120°) B1 (240°)
- (b)  $\tan \theta = -\sqrt{3}$
- $\theta = 120^\circ, 300^\circ$  B1, B1
- (c)  $2\theta = 30^\circ, 150^\circ, 390^\circ, 510^\circ$  B1 (one value)  
 $\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$  B1 (2 values)  
B1 (2 values)

3.

(a) Sine rule  $\frac{10}{\sin 45^\circ} = \frac{12}{\sin \hat{A}CB}$  M1 (correct use of sine rule)

$$\sin \hat{A}CB = \frac{12 \sin 45^\circ}{10}$$

$$\hat{A}CB = 58.05^\circ \text{ or } 121.95^\circ$$

A1 (one value)

$$\hat{A}BC = 180^\circ - (45^\circ + 58.05^\circ) = 76.95^\circ$$

A1 (F.T. one slip)

$$\text{or } 180^\circ - (45^\circ + 121.95^\circ) = 13.05^\circ$$

A1

(b) Area =  $\frac{1}{2} \times 12 \times 10 \sin 76.95^\circ \approx 58.4 \text{ cm}^2$  (58.4 – 58.5) M1 (correct formula)

$$\text{or } \frac{1}{2} \times 12 \times 10 \sin 13.05^\circ \approx 13.4 \text{ cm}^2$$

A1 (both)

(F.T. candidate values)

6

4. (a)  $n$ th term =  $ar^{n-1}$  B1

$$\text{Let } S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} \quad (1)$$

B1 (at least 3 terms, one at each end)

$$rS_n = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n \quad (2)$$

M1 (multiplication by  $r$  and subtract)

$$(1) - (2)$$

$$S_n - rS_n = a - ar^n$$

$$S_n (1 - r) = a (1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

A1 (convincing)



- (b) (i)  $ar^3 = 2, ar^6 = 54$   
 $r^3 = 27$   
 $r = 3$  B1 (both)  
M1 (eliminate  $a$ )  
A1
- (ii)  $a = \frac{2}{3^3} = \frac{2}{27}$  B1 (F.T. one slip)
- $S_{10} = \frac{2}{27} \frac{(1-3^{10})}{1-3} \approx 2187.0$  M1 (correct formula,  
F.T. candidate values)  
A1
- (iii)  $\frac{2}{27} 3^{n-1} = 125000$  B1 (F.T. candidate values)
- $\therefore 3^{n-1} = 1687500$   
 $(n-1) \ln 3 = \ln 1687500$  M1 (attempt to take logs)  
 $n = 1 + 13.05 = 14.05$  A1 (C.A.O.)  
Least value is 15 A1 (F.T. candidate's  $n$ )

14

5. (a)  $2a + d = 3$  (1)  
 $a + 7d = 47$  (2)  
Solve (1), (2)  $d = 7, a = -2$  B1 ( $a + a + d = 3$ )  
B1  
M1 (attempt to solve)  
A1 (C.A.O.)
- (b)  $S_{20} = \frac{20}{2} [2 \times -2 + 19 \times 7]$  M1 (correct formula with  
candidate values)  
 $= 1290$  A1 (F.T. candidate values)

6

6.  $\frac{5x^{\frac{4}{3}}}{4} + \frac{3x^{-2}}{-2} (+C)$   
 $\frac{5}{3}$

B1, B1

$\left( \frac{15}{4} x^{\frac{4}{3}} - \frac{3}{2x^2} \right)$

2

7. (a) B  $y = 0, 4 - x^2 = 0$  M1 (setting  $y = 0$ )

$x = \pm 2$

$B(2, 0)$

A1

A  $4 - x^2 = 3x$   
 $x^2 + 3x - 4 = 0$   
 $x = -4, 1$   
 $A(1, 3)$

M1 (equating  $ys$ )

M1 (correct attempt to solve quad)

A1

$$\begin{aligned}
 (b) \quad \text{Area} &= \int_0^1 3x dx + \int_1^2 (4 - x^2) dx \\
 &= \left[ \frac{3x^2}{2} \right]_0^1 + \left[ 4x - \frac{x^3}{3} \right]_1^2 \\
 &= \frac{3}{2} + 8 - \frac{8}{3} - 4 + \frac{1}{3} \\
 &= \frac{19}{6}
 \end{aligned}$$

M1 (use of integration to find area)  
M1 (addition of areas)  
B3 (integration)  
M1 (use of candidate's limits, any order)  
A1 (C.A.O.)

12

8. (a) Centre (4, -2)

Radius =  $\sqrt{4^2 + (-2)^2} = 3$

(b) Centre is (0, 0), radius =  $a$

Distance between centres =  $\sqrt{4^2 + 2^2} = \sqrt{20}$

Circles touch if

$$\sqrt{20} = a + 3$$

$$a \approx 1.47$$

B1  
M1 (correct method of finding radius)  
A1  
B1 (both)  
B1  
M1 (F.T. distance)  
A1 (F.T. distance)

7

9. (a)  $\frac{1}{2} 4^2 (\theta + \phi) = 15.2$

$$\theta + \phi = \frac{15.2}{8} = 1.9 \quad (1)$$

(b)  $4\theta - 4\phi = 3.2$

$$\theta - \phi = 0.8 \quad (2)$$

Solve (1), (2)

$$\theta = 1.35, \phi = 0.55$$

M1 (use of correct formula)  
A1 (convincing)  
M1 (use of correct formula)  
A1  
M1 (attempt to solve)  
A1 (F.T. one slip)

6

10. (a) Let  $x = a^p$ ,  $y = a^q$   
 $\log_a x = p$ ,  $\log_a y = q$   
 $xy = a^{p+q} = a^{p+q}$   
 $\log_a (xy) = p + q = \log_a x + \log_a y$   
 (b)  $\int_1^3 \log_{10} x dx + \int_1^3 \log_{10} 10 dx$   
 $\approx 0.5628 + \int_1^3 1 dx$   
 $= 0.5628 + [x]^3$ ,  
 $= 0.5628 + 3 - 1$   
 $= 2.5628$

B1 (properties of  
 $\log_a x$  and  $x = a^p$ )  
 B1 (laws of indices)

B1 (convincing)

B1 laws of logs)

B1

B1 (integration)

B1

# MATHEMATICS C3

1. (a)  $h = 0.25$
- Integral  $\approx \frac{0.25}{3} [1.7320508 + 4.2426407 + 4(2.1074644 + 3.3732634) + 2(2.6575365)]$
- $\approx 2.768$
- M1 ( $h = 0.25$  use of correct formula)  
B1 (3 values)  
B1 (2 further values)  
A1 (F.T. one slip)  
**4**
2. (a)  $\theta = 30^\circ$ , for example  
 $\tan 2\theta = \sqrt{3} \approx 1.732$
- $2 \tan \theta = \frac{2}{3} \approx 1.15$
- ( $\therefore \tan 2\theta \neq 2 \tan \theta$ )
- (b)  $4 (\operatorname{cosec}^2 \theta - 1) = 11 - 4 \operatorname{cosec} \theta$
- $4 \operatorname{cosec}^2 \theta + 4 \operatorname{cosec} \theta - 15 = 0$
- $(2 \operatorname{cosec} \theta - 3)(2 \operatorname{cosec} \theta + 5) = 0$
- $\operatorname{cosec} \theta = \frac{3}{2}, -\frac{5}{2}$
- $\sin \theta = \frac{2}{3}, -\frac{2}{5}$
- $\theta = 41.8^\circ, 138.2^\circ, 203.6^\circ, 336.4^\circ$
- M1 (substitution of  $\theta$  in both, one correct value)  
A1 (both correct and clearly unequal)  
M1 (correct use of  $\operatorname{cosec}^2 \theta = 1 + \cos^2 \theta$ )  
M1 (grouping terms and attempting to solve quad. in  $\operatorname{cosec} \theta$  or  $\sin \theta$  correct formula or  $(a \operatorname{cosec}^2 \theta + b)(\operatorname{cosec} \theta + d)$  where  $ac = \text{coefft of } \operatorname{cosec}^2 \theta$   $bd = \text{constant term}$ )  
A1 (CAO)  
B1 ( $41.5^\circ - 42^\circ$ )  
B1 ( $2035^\circ - 204^\circ$ )  
B1 ( $336^\circ - 336.5^\circ$ )
3. (a)  $4y^3 \frac{dy}{dx} + x^3 \frac{dy}{dx} + 3x^2 y = 2x + 4$
- $4 \frac{dy}{dx} + 8 \frac{dy}{dx} + 12 = 8$
- $\frac{dy}{dx} = -\frac{4}{12}$  (o.e.)
- B1 ( $4y^3 \frac{dy}{dx}$ )  
B1 ( $x^3 \frac{dy}{dx} + 3x^2 y$ )  
B1 ( $2x + 4$ )  
B1 (C.A.O.)

(b) (i)  $\frac{dy}{dx} = \frac{12t^3}{6t^2}$  (o.e.) M1 A1

(ii)  $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{2}{6t^2}$  (o.e.) M1 (use of correct formula)

A1 (F.T. one slip)

8

4. (a)  $\frac{2}{1+x^2} - \frac{12x}{1+x^2} - 8x = 0$  B1  $\left(\frac{2}{1+x^2}\right)$   
M1  $\left(\frac{kx}{1+x^2}\right)$  A1 ( $k = 12$ )

$2 - 12x - 8x - 8x^3 = 0$

A1 (correct unsimplified equation, F.T. one slip)

$4x^3 + 10x - 1 = 0$

A1 (convincing)

(b)  $\frac{x}{0} \frac{4x^3 + 10x - 1}{1 \quad -1 \quad 13}$  Change of sign indicates  
presence of root

M1 (attempt to find signs or values)

A1 (correct signs, values and conclusion)

$x_0 = 0.1, x_1 = 0.0996, x_2 = 0.0996048$

B1 ( $x_1$ )

$x_3 = 0.0996047 \approx 0.099605$

B1 ( $x_3$ , rounded or unrounded)

Check 0.0996045, 0.0996055

M1 (attempt to find signs or values)  
A1 (correct)

|                       |                                    |
|-----------------------|------------------------------------|
| $\frac{x}{0.0996045}$ | $\frac{4x^3 + 10x - 1}{-0.000002}$ |
| $0.0996055$           | $0.000008$                         |

Root lies between 0.0996045 and 0.0996055 and is therefore 0.099605 to 6 decimal places

A1 (correct conclusion)

12

5. (a)  $-e^{3x} \sin x + 3e^{3x} \cos x$

M1 ( $f(x)e^{3x} + g(x) \cos x$ )  
A1 ( $f(x) = -\sin x, g(x) = ke^{3x}$ )  
A1 ( $k = 3$ , correct unambiguous answer)

- (b)  $\frac{(3x^2 + 2)4x - (2x^2 + 1)6x}{(3x^2 + 2)^2}$   
 $= \frac{2x}{(3x^2 + 2)^2}$   
 M1  $\left( \frac{(3x^2 + 2)f(x) - (2x^2 + 1)g(x)}{(3x^2 + 2)^2} \right)$   
 A1 ( $f(x) = 4x, g(x) = 6x$ )  
 A1 (C.A.O.)
- (c)  $10x \sec^2(5x^2 + 3)$   
 M1 ( $\sec^2(5x + 3)$ , allow  $g(x) = 1$ )  
 A1 (correct unambiguous answer)
- (d)  $\frac{1}{2x} \times 2 = \frac{1}{x}$   
 M1 ( $\frac{k}{2x}$ , allow  $k = 1, 2$ )  
 A1 (simplified answer)
- (e)  $\frac{3}{\sqrt{1 - (3x)^2}} \left( = \frac{3}{\sqrt{1 - 9x^2}} \right)$   
 M1 ( $\frac{k}{\sqrt{1 - (3x)^2}}$  (o.e.), allow  $k = 1$ )  
 A1 ( $k = 3$ )

12

6. (a)  $3x - 8 \leq 5$   
 $x \leq \frac{13}{3}$   
 $3x - 8 \geq -5$   
 $x \geq 1$   
 $1 \leq x \leq \frac{3}{13}$  (or  $x \geq 1$  and  $x \leq \frac{3}{13}$ )  
 B1  
 M1  
 A1 (must indicate both conditions apply)
- (b) Graphs  
 M1 (for  $|x|$ , V shape through origin)  
 A1 (translation in +ve y direction, cusp at  $(\pm 2, 1)$ )  
 A1 (cusp at  $(-2, 1)$ )  
 A1 (correct relative positions)

7

7. (a) (i)  $\frac{4}{7} \ln |7x + 2| - \frac{5}{6(3x + 1)^2}$   
 M1 ( $k \ln(7x + 2)$ )  
 (+C)  
 A1  $\left( k = \frac{4}{7} \right)$   
 M1  $\left( \frac{k}{(3x + 1)^2} \right)$  A1  $\left( k = -\frac{5}{6} \text{ (o.e.)} \right)$

$$(ii) \quad \frac{1}{2} \sin 2x (+C)$$

$$M1 \ (k \sin 2x, k = \frac{1}{2}, -\frac{1}{2}, 1, 2)$$

$$A1 \ (k = \frac{1}{2})$$

$$(b) \quad \left[ 2e^{\frac{x}{2}} \right]_0^4$$

$$= 2e^2 - 2e^0$$

$$\sim 12.8$$

$$M1 \ (ke^{\frac{x}{2}}, k = 2, \frac{1}{2}, 1)$$

$$A1 \ (k = 2)$$

$$M1 \ (ke^2 - ke^0, \text{allowable } ks)$$

$$A1 \ (\text{C.A.O. at least 3 sig. figs})$$

10

$$8. \quad (a) \quad \text{Let } y = 3x^2 + 4$$

$$x^2 = \frac{y-4}{3}$$

$$x = \pm \sqrt{\frac{y-4}{3}}$$

$$M1 \ (x^2 = f(y))$$

Choose +  $\because$  domain of  $f$  is  $x \geq 0$

$$A1$$

$$x = \sqrt{\frac{y-4}{3}}$$

$$A1 \ (\text{F.t. one slip})$$

$$f^{-1}(x) = \sqrt{\frac{x-4}{3}}$$

$$A1 \ (\text{F.T. one slip})$$

Domain is  $x \geq 4$ , range is  $f^{-1}(x) \geq 0$  (o.e.)

$$B1, B1$$

Domain is  $x \geq 4$ , range is  $f^{-1}(x) > 0$

(b) Graphs

M1 (full or half parabola passing through (0,b))

A1 (r.h. branch and minimum at (0,4))

A1 F.T. b, full or half parabola))

9

$$9. \quad (a) \quad \text{Domain is } (2, \infty)$$

$$B1$$

$$(b) \quad (fg(x) = 5)$$

$$e^{\ln(x^2-4)} = 5$$

$$= 5$$

M1 (correct order)

$$x^2 - 4 = 5$$

or

$$\ln(x^2 - 4) = \ln 5$$

$$A1 \ (\text{either})$$

$$x^2 = 9$$

$$x = 3 \ (-3 \text{ not in domain})$$

$$A1$$

$$A1 \ (\text{with reason})$$

5

# MATHEMATICS FP1

1.  $\text{Mod} = \sqrt{3+1} = 2$  B1
- $\text{Arg} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} \quad (30^\circ)$  B1
- $\sqrt{3} + i = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$  B1
- $\text{Arg of } (\sqrt{3} + i)^n = \frac{n\pi}{6}$  M1A1
- Power is real when
- $\frac{n\pi}{6}$  is an integer multiple of  $\pi$  A1
- Smallest  $n = 6$ . A1
2.  $\text{Det} = 1.2 + 2\lambda + \lambda(3\lambda - 4)$  M1m1A1
- $= 3\lambda^2 - 2\lambda + 2$  A1
- EITHER  $= 3((\lambda - 1/3)^2 + 5/9)$  M1
- $> 0$  for all  $\lambda$  A1
- OR  $'b^2 - 4ac' = -20$  M1
- So determinant not equal to zero for any real  $\lambda$ . A1
3.  $f(x+h) - f(x) = \frac{1}{1-(x+h)^2} - \frac{1}{1-x^2}$  M1A1
- $= \frac{1-x^2 - [1-(x+h)^2]}{[1-(x+h)^2](1-x^2)}$  m1
- $= \frac{h(2x+h)}{[1-(x+h)^2](1-x^2)}$  A1
- $f'(x) = \lim_{h \rightarrow 0} \frac{(2x+h)}{[1-(x+h)^2](1-x^2)}$  M1
- $= \frac{2x}{(1-x^2)^2}$  A1
4.  $\frac{11+7i}{1+i} = \frac{(11+7i)(1-i)}{(1+i)(1-i)}$  M1A1
- $= \frac{18-4i}{2} = 9-2i$  A1
- $2(x+iy) + (x-iy) = 9-2i$  M1
- Equating real and imaginary parts m1
- $x = 3$  and  $y = -2$  A1



5. (a)  $\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$  B1

$\mathbf{T}_2 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  B1

$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$  M1

$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  A2

Note: Multiplying the wrong way gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Award M1A1}$$

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{M1}$$

giving  $-y - 2 = x$  and  $x + 1 = y$

Fixed point is  $(-3/2, -1/2)$ .

FT from wrong answer to (a) – the incorrect matrix above leads to  $(-1/2, 3/2)$ .

A1  
M1A1

6. (a) Assume the proposition is true for  $n = k$ , that is

$$\sum_{r=1}^k (2r+1) = (k+1)^2 \quad \text{B1}$$

Consider

$$\begin{aligned} \sum_{r=1}^{k+1} (2r+1) &= (k+1)^2 + 2k + 3 \\ &= (k+2)^2 \end{aligned} \quad \text{M1A1} \quad \text{A1}$$

So, if the proposition is true for  $n = k$ , it is also true for  $n = k + 1$ .

A1

(b) Since  $3 \neq 4$ , P is false for  $n = 1$  is therefore false.

B2

7. (a) Using reduction to echelon form,
- $$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & \lambda - 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu - 4 \end{bmatrix}$$
- M1A1A1
- $$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & \lambda - 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu - 10 \end{bmatrix}$$
- A1
- The solution will not be unique if  $\lambda = 3$ . A1
- (b) The equations are consistent if  $\mu = 10$ . B1  
Put  $z = \alpha$  M1  
 $y = \alpha - 6$  A1  
 $x = 16 - 4\alpha$  M1A1
8. (a) Let the roots be  $\alpha - 1, \alpha, \alpha + 1$ . M1  
Then  $\alpha(\alpha - 1) + \alpha(\alpha + 1) + (\alpha - 1)(\alpha + 1) = 47$  m1  
 $\alpha^2 = 16$  A1  
 $\alpha = -4$  A1  
The roots are  $-3, -4$  and  $-5$ . A1
- (b)  $p = 12, q = 60$  B1B1
9. (a)  $\ln f(x) = \frac{1}{x} \ln(x)$  M1  
 $\frac{f'(x)}{f(x)} = -\frac{1}{x^2} \ln(x) + \frac{1}{x^2}$  m1A1  
 $= \frac{1}{x^2} (1 - \ln(x))$  A1  
At the stationary point,  
 $\ln(x) = 1$  M1  
so  $x = e$  (2.72) and  $y = e^{\frac{1}{e}}$  (1.44) A1A1
- (b) We now need to determine its nature.  
We see from above that  
For  $x < e$ ,  $f'(x) > 0$  and for  $x > e$ ,  $f'(x) < 0$  M1  
Showing it to be a maximum. A1

10.

$$w = \frac{z+3}{z+1}$$

$$wz + w = z + 3 \quad \text{M1}$$

$$z(w-1) = 3-w \quad \text{A1}$$

$$z = \frac{3-w}{w-1} \quad \text{A1}$$

Since  $|z| = 1$ , it follows that

$$|3-w| = |w-1| \quad \text{M1}$$

$$\sqrt{(3-u)^2 + v^2} = \sqrt{(u-1)^2 + v^2} \quad \text{A1}$$

$$9 - 6u + u^2 + v^2 = u^2 - 2u + 1 + v^2 \quad \text{A1}$$

leading to  $u = 2$ . A1

Straight line parallel to the  $v$ -axis (passing through (2,0)). B1

#### ALTERNATIVE METHOD

$$u + iv = \frac{x+3+iy}{x+1+iy} \cdot \frac{x+1-iy}{x+1-iy} \quad \text{M1}$$

$$= \frac{(x+1)(x+3) + y^2 + iy(x+1-x-3)}{(x+1)^2 + y^2} \quad \text{A1}$$

Equating real and imaginary parts, M1

$$u = \frac{x^2 + y^2 + 4x + 3}{x^2 + y^2 + 2x + 1} \quad \text{A1}$$

$$v = \frac{-2y}{x^2 + y^2 + 2x + 1} \quad \text{A1}$$

Putting  $x^2 + y^2 = 1$ , M1

$$u = 2 \quad \text{A1}$$

which is a straight line parallel to the  $v$ -axis (passing through (2,0)). B1

# MATHEMATICS M1

1(a) (i) Use of  $v^2 = u^2 + 2as$  with  $u = 0$ ,  $a = (\pm)9.8$ ,  $s = (\pm)160$  M1  
 $v^2 = 0 + 2 \times 9.8 \times 160$  A1

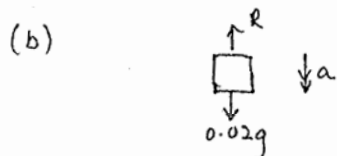
$v = \underline{56 \text{ ms}^{-1}}$  A1

(ii) Use of  $s = ut + \frac{1}{2}at^2$  with  $s = (\pm)160$ ,  $u = 0$ ,  $a = (\pm)9.8$  M1

$(-160) = \frac{1}{2} \times (-)9.8 t^2$  ft v A1✓

$t = \underline{\frac{40}{7} \text{ s}} = 5\frac{5}{7} \approx 5.71$

too A1



use of 0.2 for mass make as 0.02

(i) Use of N2L

dim. correct M1

$0.02g - 0.096 = 0.02a$  A1

$a = \underline{5 \text{ ms}^{-2}}$  A1

(iii) Use of  $s = ut + \frac{1}{2}at^2$  with  $u = 0$ ,  $a = (\pm)5$ ,  $t = 4$  M1

$s = \frac{1}{2} \times 5 \times 4^2$  ft a A1✓

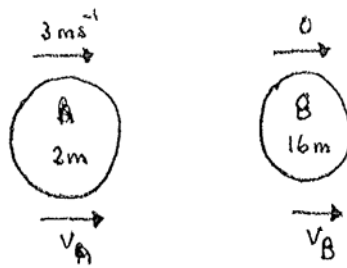
$= 40$

$\therefore \text{Height above ground} = 160 - 40$

$= \underline{120 \text{ m}}$

A1✓

2(a)



Conservation of momentum

$$2 \times 3(m) + 16 \times 0(m) = 2v_A(m) + 16v_B(m)$$

$$v_A + 8v_B = 3$$

allow min 2 eqs M1

A1

Restitution

$$v_B - v_A = -\frac{1}{2}(0 - 3)$$

$$-v_A + v_B = \frac{3}{2}$$

allow 1 mark M1

A1

Adding

$$9v_B = \frac{9}{2}$$

$$v_B = \frac{1}{2} \text{ ms}^{-1}$$

$$v_A = v_B - \frac{3}{2}$$

$$= -1 \text{ ms}^{-1}$$

dep on both Ms M1

ft 1 slip A1

ft 1 slip A1

Allow v<sub>A</sub> going to left.  
consistent equations

(b) Impulse I = 2.m (-1 - 3)

$$= -8m \text{ NS} = \text{kg m s}^{-1}$$

M1

allow no -

A1 B1

In the direction <sup>opposite to</sup> the original motion of A

A1

3 (a)

$$T = \frac{600}{20}$$

$$= \underline{30}$$

BI

(b) using  $s = ut + \frac{1}{2}at^2$  with  $u = 15$ ,  $s = 600$ ,  $t = 30$  M1

$$600 = 15 \times 30 + \frac{1}{2}a \times 30^2$$

AI

$$a = \underline{\frac{1}{3} \text{ ms}^{-2}}$$

accept 0.3, 0.33 etc

AI

(c) using  $s = \frac{1}{2}(u+v)t$  with  $s = 600$ ,  $u = 15$ ,  $t = 30$  M1

$$600 = \frac{1}{2}(15+v) \times 30$$

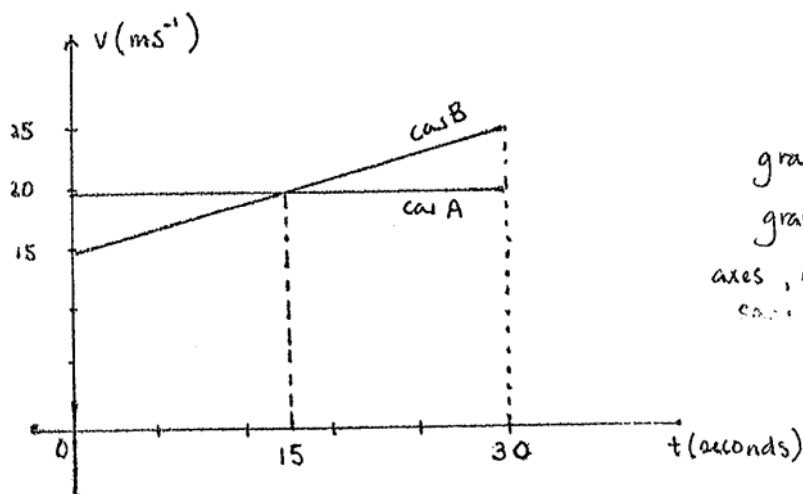
AI

$$v = \underline{25 \text{ ms}^{-1}}$$

CAO AI

PH of 0.3 or 0.33 used

(d)

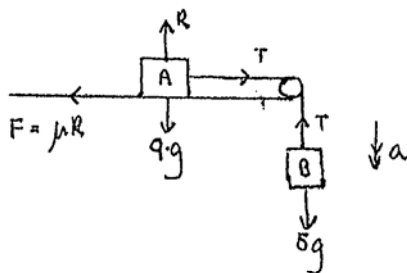


graph A BI allow any graph  
graph B BI allow any graph  
axes, scales etc BI  
correct graph

required time when A and B<sup>A</sup> have the same speed is 15 s

ft c's v BI

4 (a)



N2L to B

$$T = 5g = 49$$

81

N2L to A

$$T = F = 49$$

81

Resolve  $\uparrow$  for A

$$R = 9g = 88.2$$

81

System in equilibrium

$$F \leq \mu R$$

$$\mu R = \mu R \times 0.216$$

M1

$$\mu \geq \frac{5g}{9g} = \frac{5}{9}$$

$$\text{and } a = 0$$

(condition)

A1

(b)

$$R = 9g = 88.2$$

$$F = \mu R$$

$$= 0.5 \times 9 \times 9.8$$

$$= 4.5g = 44.1$$

81

N2L to B

$$5g - T = 5a$$

M1 A1

N2L to A

$$T - F = 9a$$

M1 A1

$$T - 4.5g = 9a$$

Adding

$$5g - 4.5g = 14a$$

dep on both Ms. m1

$$a = \frac{0.5 \times 9.8}{14}$$

$$= \underline{0.35 \text{ ms}^{-2}}$$

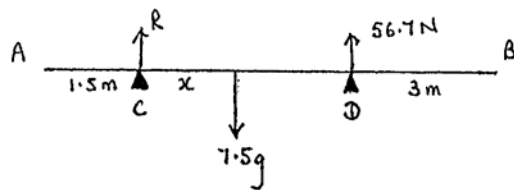
A1

$$T = 9 \times 0.35 + 4.5 \times 9.8$$

$$= \underline{47.25 \text{ N}}$$

dep. only on one M  
Cao A1

5.



(a) Moment about C

$$7.5g \times x = 56.7 \times (5 - 1.5)$$

$$x = \underline{2.7 \text{ m.}}$$

dim. correct

M1

A1 B1

cao A1

(b) Resolve  $\uparrow$

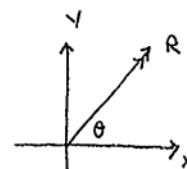
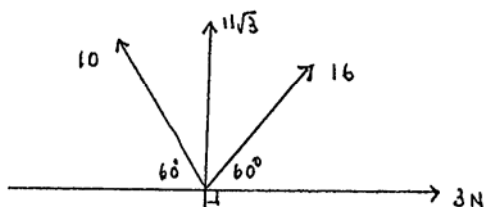
$$R + 56.7 = 7.5g$$

$$R = \underline{16.8 \text{ N}}$$

M1

A1

6.



Resolve  $\rightarrow$

$$x = 3 + 16 \cos 60^\circ - 10 \cos 60^\circ$$

$$= \underline{6 \text{ N}}$$

resolution

M1

A1 all f.e.  
(not f.e.)  
A1

Resolve  $\uparrow$

$$y = 11\sqrt{3} + 10 \sin 60^\circ + 16 \sin 60^\circ$$

$$= \underline{24\sqrt{3} \text{ N}} = 41.5692$$

resolution

M1

A1 all f.e.  
(not f.e.)  
A1

$$R = \sqrt{6^2 + (24\sqrt{3})^2}$$

$$= \underline{42 \text{ N}}$$

M1

ft x, y  
penalise PA.

A1✓

$$\tan \theta = \frac{24\sqrt{3}}{6}$$

M1

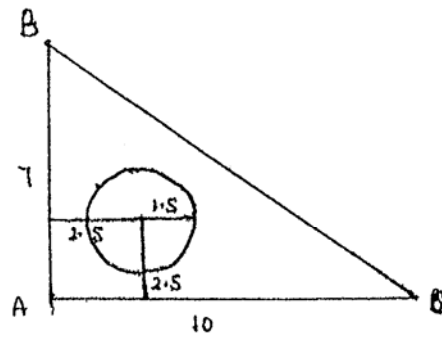
$$\theta = \underline{81.8^\circ}$$

ft x, y.

A1✓



7.



|     |              |                                          | from AB        | from AC       |                   |
|-----|--------------|------------------------------------------|----------------|---------------|-------------------|
| (a) | $\Delta ABC$ | 35                                       | $\frac{10}{3}$ | $\frac{7}{3}$ | BI BI BI<br>Areas |
|     | Circle       | $\pi \left(\frac{3}{2}\right)^2 = 7.069$ | $\frac{5}{2}$  | $\frac{5}{2}$ | BI                |
|     | Lamina       | $35 - \frac{9}{4}\pi = 27.92$            | x              | y             |                   |

$$(i) \quad 35 \times \frac{10}{3} = \frac{9}{4}\pi \times \frac{5}{2} + (35 - \frac{9}{4}\pi)x$$

$$x = \underline{3.54}$$

MI AI  
COO AI

$$(ii) \quad 35 \times \frac{7}{3} = \frac{9}{4}\pi \times \frac{5}{2} + (35 - \frac{9}{4}\pi)y$$

$$y = \underline{2.29}$$

MI AI  
COO AI

(b) Required angle  $\theta = \tan^{-1}\left(\frac{2.29}{3.54}\right)$

$$= \underline{32.9^\circ}$$

MI  
ft x, y AI

**M1 – ASSESSMENT OBJECTIVES**

| <b>Q.No.</b> | <b>AO1</b> | <b>AO2</b> | <b>AO3</b> | <b>AO4</b> | <b>AO5</b> | <b>Total</b> |
|--------------|------------|------------|------------|------------|------------|--------------|
| <b>1</b>     | <b>2</b>   | <b>2</b>   | <b>6</b>   | <b>2</b>   |            | <b>12</b>    |
| <b>2</b>     | <b>3</b>   | <b>2</b>   | <b>4</b>   | <b>2</b>   |            | <b>11</b>    |
| <b>3</b>     | <b>2</b>   | <b>3</b>   | <b>4</b>   | <b>2</b>   |            | <b>11</b>    |
| <b>4</b>     | <b>3</b>   | <b>3</b>   | <b>4</b>   | <b>1</b>   | <b>2</b>   | <b>13</b>    |
| <b>5</b>     | <b>1</b>   | <b>1</b>   | <b>2</b>   | <b>2</b>   |            | <b>6</b>     |
| <b>6</b>     | <b>2</b>   | <b>2</b>   | <b>2</b>   | <b>1</b>   | <b>3</b>   | <b>10</b>    |
| <b>7</b>     | <b>3</b>   | <b>3</b>   | <b>3</b>   | <b>2</b>   | <b>1</b>   | <b>12</b>    |
|              |            |            |            |            |            |              |
| <b>TOTAL</b> | <b>16</b>  | <b>16</b>  | <b>25</b>  | <b>12</b>  | <b>6</b>   | <b>75</b>    |

# MATHEMATICS S1

1. (a)  $P(\text{Score at least 3 on one die}) = \frac{4}{6}$  B1

$P(\text{Score at least 3 on both dice}) = \left(\frac{4}{6}\right)^2 = \frac{4}{9}$  M1A1

[Award B1 for the  $6 \times 6$  array of possible scores, M1 for counting the number of these satisfying the required condition and A1 for the correct answer]

(b) Possibilities are 6,3;5,2;4,1 and reverse, ie 6 possibilities M1A1

$$\text{Prob} = \frac{6}{36} \left(\frac{1}{6}\right) (\text{cao})$$
 A1

2. (a)  $P(A \cup B) = P(A) + P(B)$  gives  $P(B) = 0.2$  M1A1

(b)  $0.5P(B) = 0.5 + P(B) - 0.7$  M1A1  
 $0.5P(B) = 0.2$  gives  $P(B) = 0.4$  M1A1

(c)  $0.5 \times 0.3 = 0.5 + P(B) - 0.7$  M1A1  
 $P(B) = 0.35$  A1

3. (a) 2. B1

(b) (i)  $\text{Prob} = e^{-4} \cdot \frac{4^3}{3!} = 0.195$  1A1

[Accept  $0.7619 - 0.5665$  or  $0.4335 - 0.2381$  from tables, M1 for at least 1 correct value]

(ii)  $\text{Prob} = 0.9084 - 0.1107$  or  $0.8893 - 0.0916 = 0.7977$  (cao) B1B1B1

(c)  $E(C) = 5 + 4 \times 4 = 21$  M1A1  
 $SD = \sqrt{16 \times \text{Var}(X)} = 8$  [Award M1 for 64] M1A1

4. (a)  $\text{Prob} = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$  or  $\frac{\binom{5}{3}}{\binom{8}{3}} = \frac{5}{28}$  M1A1

$$(b) \quad P(3 \text{ blue}) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \text{ or } \frac{\binom{3}{3}}{\binom{8}{3}} \quad (= \frac{1}{56}) \quad \text{B1}$$

$$P(2 \text{ blue}) = 3 \times \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} \text{ or } \frac{\binom{3}{2} \times \binom{5}{1}}{\binom{8}{3}} \quad (= \frac{15}{56}) \quad \text{M1A1}$$

$$\text{Reqd prob} = \frac{16}{56} \quad (\frac{2}{7}) \quad \text{B1}$$

5.  $np = 20$  ;  $np(1-p) = 16$  M1A1A1  
Valid attempt to solve equations (FT) M1  
 $p = 0.2$  B1  
 $n = 100$  B1

6. (a)  $P(4) = \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{4}$  M1A1  
 $= \frac{5}{24}$  A1

(b)  $P(\text{cube} | 4) = \frac{1/12}{5/24} = \frac{2}{5} \text{ (cao)}$  B1B1B1

7. (a) Number of defective glasses  $X$  is  $B(24, 0.05)$  (si) B1  
 $P(X=2) = \binom{24}{2} \times 0.05^2 \times 0.95^{22} = 0.223$  M1A1

(b)  $P(3 \leq X \leq 5) = 0.9622 - 0.5405 \text{ or } 0.4595 - 0.0378 = 0.422 \text{ (cao)}$  B1B1B1

(c) Number of defective glasses is now  $B(120, 0.05) \approx \text{Poi}(6)$  B1  
 $P(Y < 8) = 0.744 \text{ (or } 1 - 0.256)$  M1A1

8. (a)  $[0, 0.7]$  [Accept (0.0.7)] B1B1  
[Award B1 for  $\{0, 0.1, \dots, 0.7\}$  or similar]

(b) (i)  $E(X) = 0.1 + 0.4 + 3\theta + 4(0.7 - \theta) = 3.3 - \theta$  M1A1  
 $3.3 - \theta = 3$  gives  $\theta = 0.3$  M1A1

(ii)  $E(X^3) = 0.1 \times 1 + 0.2 \times 8 + 0.3 \times 27 + 0.4 \times 64$  M1A1  
 $= 35.4$  A1

|    |     |       |                                                                                |      |
|----|-----|-------|--------------------------------------------------------------------------------|------|
| 9. | (a) | (i)   | $\int_1^4 kx^2 dx = \frac{k}{3} [x^3]_1^4 = 21k$                               | M1A1 |
|    |     |       | Int = 1 gives $k = \frac{1}{21}$                                               | A1   |
|    |     | (ii)  | $E(X) = \frac{1}{21} \int_1^4 x \cdot x^2 dx$ (Limits required, here or later) | M1   |
|    |     |       | $= \frac{1}{21} \cdot \frac{1}{4} [x^4]_1^4$                                   | B1   |
|    |     |       | $= \frac{85}{28} \text{ (3.04)}$                                               | A1   |
|    | (b) | (i)   | $F(x) = \frac{1}{21} \int_1^x y^2 dy$                                          | M1   |
|    |     |       | $= \frac{1}{63} [y^3]_1^x$                                                     | A1   |
|    |     |       | $= \frac{1}{63} (x^3 - 1)$                                                     | A1   |
|    |     | (ii)  | Prob = $F(3) - F(2)$                                                           | M1   |
|    |     |       | $= \frac{19}{63} \text{ (0.302)}$                                              | A1   |
|    |     | (iii) | The median $m$ satisfies                                                       |      |
|    |     |       | $\frac{1}{63} (m^3 - 1) = 0.5$                                                 | M1   |
|    |     |       | $m^3 = 32.5$                                                                   | A1   |
|    |     |       | $m = 3.19 \text{ (cao)}$                                                       | A1   |

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