

973/01

**MATHEMATICS C1**

**Pure Mathematics**

P.M. TUESDAY, 10 January 2006

(1  $\frac{1}{2}$  hours)

## **NEW SPECIFICATION**

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

### **INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Calculators are **not** allowed for this paper.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points  $A$ ,  $B$ ,  $C$  have coordinates  $(-2, -3)$ ,  $(6, 1)$  and  $(k, 3)$  respectively. The line  $AB$  is perpendicular to  $BC$ .

(a) Find the gradient of  $AB$ . [2]

(b) Show that  $k = 5$ . [3]

(c) The line  $L$  is parallel to  $BC$  and passes through  $A$ . Find the equation of  $L$ . [2]

(d) The line  $L$  intersects the  $y$ -axis at  $D$ . Calculate the length of  $CD$ . [3]

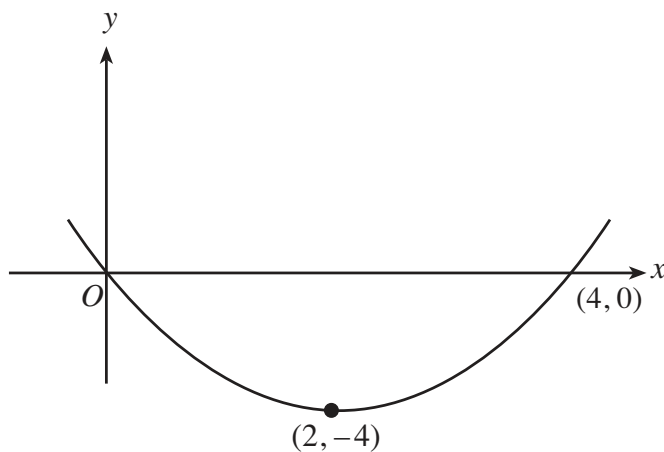
2. (a) Simplify the following.

$$\sqrt{48} + \sqrt{27} - \frac{6}{\sqrt{3}} \quad [4]$$

(b) Simplify  $\frac{2 + \sqrt{7}}{3 + \sqrt{7}}$ , expressing your answer in surd form. [4]

3. Find the equation of the normal to the curve  $y = 4x^2 - 7x + 2$  at the point  $(2, 4)$ . [4]

4.



The diagram shows the graph of  $y = f(x)$ . The curve passes through the origin, the point  $(4, 0)$  and has a minimum point at  $(2, -4)$ . Sketch on separate diagrams the graphs of

(a)  $y = -f(x)$ , [2]

(b)  $y = f(x - 2)$ , [3]

in each case giving the coordinates of the points of intersection of the graph with the  $x$ -axis and the coordinates of the stationary point.

5. Given that the quadratic equation

$$(k + 2)x^2 + 4x + k + 5 = 0$$

has no real roots, show that

$$k^2 + 7k + 6 > 0.$$

Find the range of values of  $k$  satisfying this inequality.

[7]

6. (a) Given that when the polynomial  $ax^3 - x^2 - 7x + 6$  is divided by  $x - 2$  the remainder is 4, show that  $a = 2$ . [2]

- (b) Solve the equation  $2x^3 - x^2 - 7x + 6 = 0$ . [5]

7. (a) Using the binomial theorem, expand  $(3x + 2)^3$ , simplifying each term of the expansion. [3]

- (b) In the binomial expansion of  $(1 + 2x)^n$  the coefficient of  $x^2$  is twice the coefficient of  $x$ . Given that  $n > 0$ , find the value of  $n$ . [4]

8. (a) Given that  $y = 2x^2 - 5x + 3$ , find  $\frac{dy}{dx}$  from first principles. [5]

- (b) Given that  $y = \frac{a}{x} + 2x^{\frac{3}{2}}$  and  $\frac{dy}{dx} = 7$  when  $x = 4$ , find the value of the constant  $a$ . [4]

9. (a) Express  $23 + 6x - x^2$  in the form  $b - (x - a)^2$ , where the constants  $a$  and  $b$  are to be determined. Hence find the greatest value of  $23 + 6x - x^2$  and the corresponding value of  $x$ . [4]

- (b) Use the results found in (a) to deduce the least value of  $\frac{1}{30 + 6x - x^2}$ . [2]

10. The curve  $C$  has equation

$$y = 2 + 6x^2 - 2x^3.$$

- (a) Find the coordinates and the nature of each of the stationary points of  $C$ . [7]

- (b) Sketch  $C$ , indicating clearly the nature of each of the stationary points. [3]

- (c) State clearly, giving a reason, the number of real roots of the equation

$$2 + 6x^2 - 2x^3 = 0. \quad [2]$$