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WELSH JOINT EDUCATION COMMITTEE
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General Certificate of Education
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MARKING SCHEMES

JANUARY 2006

MATHEMATICS
(New Specification)

WJEC
CBAC

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

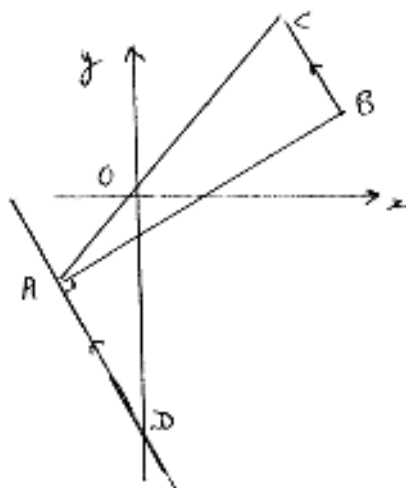
MATHEMATICS C1

1. (a) Gradient of $AB = \frac{1+3}{6+2} = \frac{1}{2}$

M1 (correct method)

A1 (grad = $\frac{1}{2}$)

(b)



Gradient of $BC = \frac{3-1}{k-6} = \frac{2}{k-6}$

M1 (use of $m_1 m_2 = -1$,
seen or implied)

Then $\frac{1}{2} \times \frac{2}{k-6} = -1$

M1 (method of finding
equation in k)

$$\begin{aligned} k-6 &= -1 \\ k &= 5 \end{aligned}$$

A1 (C.A.O.) (convincing)

(c) Gradient of $L = -2$

B1 (F.T. for same gradient as
 BC or $-\frac{1}{\text{grad} AB}$)

Equation of L is $y+3 = -2(x+2)$

B1 (give mark here)

(d) Coords of D : $x=0, y=-7$

B1(F.T. unsimplified equation of L)

$$CD = \sqrt{(5-0)^2 + (3+7)^2} = \sqrt{125}$$

M1 (correct formula)

A1 ($\sqrt{\text{single no}}$, F.T. derived
coords of D)

2. (a) $4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3}$

$= 5\sqrt{3}$

B1, B1, B1

B1 (F.T. one slip, answer of form $k\sqrt{3}$)

(b) $\frac{(2+\sqrt{7})(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{6-2\sqrt{7}+\sqrt{7}-7}{9-7}$

M1 (correct rationalising)

A1 numerator with $(\sqrt{7})^2 = 7$,
allow 2×3

A1 (denominator with no surds)

$= \frac{\sqrt{7}-1}{2}$

A1 (F.T. one slip)

8

3. $\frac{dy}{dx} = 8x - 7$

$= 9$ at $(2, 4)$

B1 (correct differentiation)

B1 (numerical result, F.T. one slip)

Gradient of normal $= -\frac{1}{9}$

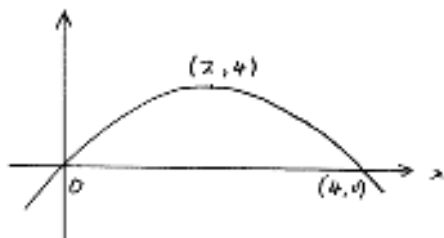
M1 $\left(\frac{-1}{\text{gradient of tgt}} \right)$

Equation is $y - 4 = -\frac{1}{9}(x - 2)$

A1 (F.T. candidate's gradient of tangent)

4

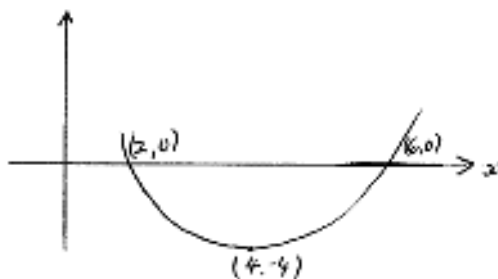
4. (a)



M1 (reflection in x - axis)

A1 (correct points)

(b)



M1 (x translation to right)

B1 (st.pt) (on diagram or stated)

A1 (points of intersection)

5

5.	For no real roots, $4^2 - 4(k+2)(k+5) < 0$ $4 - k^2 - 7k - 10 < 0$ $k^2 + 7k + 6 > 0$ $(k+6)(k+1) > 0$ $k < -6 \text{ (and/or) } k > -1$ <u>or</u> $k < -6, k > -1$ $-1 < k < -6 \text{ or } k > -1 \text{ or } k < -6$	M1 ($b^2 - 4ac$, correct b , a or c correct) A1 (correct) M1 ($b^2 - 4ac < 0$) A1 (convincing) B1 (fixed pts, $-1, -6$) B2 (F.T. fixed points) All gain B1	7
6.	(a) $f(2) = 4$ $8a - 4 - 14 + 6 = 4$ $8a = 16$ $a = 2$ (b) $f(1) = 2 - 1 - 7 + 6 = 0$ $x - 1$ is a factor $2x^3 - x^2 - 7x + 6 = (x - 1)(2x^2 + x - 6)$ $= (x - 1)(2x - 3)(x + 2)$ Roots are $1, \frac{3}{2}, -2$	M1 (remainder theorem or division with remainder equated to 4) A1 M1 (correct use of factor theorem with appropriate factor mentioned) A1 (correct factor) m1 ($2x^2 + ax + b$, a or b correct, any method) A1 (correct quad. factor) A1 (3 factors and 3 roots, F.T. one slip)	7
7.	(a) $(3x + 2)^3 = (3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + 2^3$ $= 27x^3 + 54x^2 + 36x + 8$ (b) $\frac{n(n-1)}{2}(2)^2 = 2(2n)$ $n = 3$	B3 (-1 for each error, any method) M1 (${}^nC_2 2^p = 2k{}^nC_1, k=2, \frac{1}{2}, p=1,2$) A2 ($\frac{n(n-1)}{2} 2^2 = 2(2n)$) A1 (C.A.O.)	7
8.	(a) Let $y = 2x^2 - 5x + 3$ $y + \Delta y = 2(x + \Delta x)^2 - 5(x + \Delta x) + 3$ $\Delta y = 4x\Delta x + 2(\Delta x)^2 - 5$ $\frac{\Delta y}{\Delta x} = 4x + 2\Delta x - 5$ Let $\Delta x \rightarrow 0$ $\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = 4x - 5$	B1 M1 (attempt to find Δy) A1 M1 (divide by Δx and let $\Delta x \rightarrow 0$) A1 (award only if some mention of limit, clear presentation and no abuse of notation)	

(b) $\frac{dy}{dx} = -\frac{a}{x^2} + 3x^{\frac{1}{2}}$ (o.e.) B1, B1

$$-\frac{a}{4^2} + 3 \times 4^{\frac{1}{2}} = 7$$

$$a = -16$$

M1 ($f'(4)=7$, reasonable diffn)
A1 (C.A.O.)

9

9. (a) $23 + 6x - x^2 = 32 - (x - 3)^2$

Greatest value = 32
when $x = 3$

B1 ($-(x - 3)^2$)
B1 (32)
B1 (F.T. candidate's b and a)
B1

(b) $\frac{1}{30 + 6x - x^2} = \frac{1}{7 + 23 + 6x - x^2}$ A1

Least value = $\frac{1}{39}$ A1 (F.T. b, a)

6

10. (a) $\left(\frac{dy}{dx} = 0\right)$ B1 $\left(\frac{dy}{dx}\right)$

$$12x - 6x^2 = 0$$

$$x = 0, 2$$

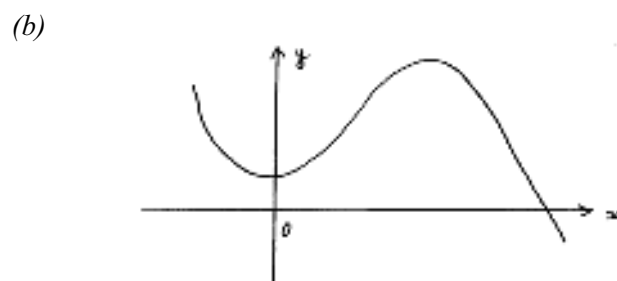
When $x = 0, y = 2$; when $x = 2, y = 10$

$$\frac{d^2y}{dx^2} = 12 - 12x$$

$$x = 0, \frac{d^2y}{dx^2} = 12 > 0 \text{ min.}$$

$$x = 2, \frac{d^2y}{dx^2} = -12 < 0 \text{ max.}$$

M1 $\left(\frac{dy}{dx} = 0\right)$
A1 (either root, F.T. one slip)
A1 (both C.A.O.)
M1 (any method)
A1 (F.T. x values)
A1



B1 (shape, negative cubic)
B1, B1 (derived stat pts)

(c) One real root as graph crosses x -axis once

M1 (reason)
A1 (F.T. graph)

12

MATHEMATICS C2

1. (a) $h = 0.2$ M1 (correct formula $h = 0.2$)
- Integral $\approx \frac{0.2}{2} [0.5 + 0.3333333 + 2(0.4980080 + 0.48449612 + 0.4512635 + 0.3980892)]$ B1 (4 values)
 ≈ 0.4497 B1 (2 values)
A1 (F.T. one slip)
- S. Case $h = \frac{1}{6}$ M1 (correct formula, $h = \frac{1}{6}$)
- Integral $\approx \frac{1}{12} [0.5 + 0.3333333 + 2(0.4988453 + 0.4909091 + 0.4705882 + 0.4354838 + 0.3877917)]$ B1 (all values)
 ≈ 0.4500 A1 (F.T. one slip)
2. (a) $4 \cos^2 \theta - \cos \theta = 2(1 - \cos^2 \theta)$ M1 (correct use of $\sin^2 \theta + \cos^2 \theta = 1$)
- $6 \cos^2 \theta - \cos \theta - 2 = 0$ M1 (attempt to solve quadratic in $\cos \theta$, correct formula)
- $(3 \cos \theta - 2)(2 \cos \theta + 1) = 0$
- $\cos \theta = \frac{2}{3}, -\frac{1}{2}$ A1 (C.A.O.)
- $\theta = 48.2^\circ, 311.8^\circ, 120^\circ, 240^\circ$ B1 (48.2, 311.8°)
B1 (120°) B1 (240°)
- (b) $\tan \theta = -\sqrt{3}$
- $\theta = 120^\circ, 300^\circ$ B1, B1
- (c) $2\theta = 30^\circ, 150^\circ, 390^\circ, 510^\circ$ B1 (one value)
 $\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$ B1 (2 values)
B1 (2 values)

3.

(a) Sine rule $\frac{10}{\sin 45^\circ} = \frac{12}{\sin \hat{A}CB}$ M1 (correct use of sine rule)

$$\sin \hat{A}CB = \frac{12 \sin 45^\circ}{10}$$

$$\hat{A}CB = 58.05^\circ \text{ or } 121.95^\circ$$

A1 (one value)

$$\hat{A}BC = 180^\circ - (45^\circ + 58.05^\circ) = 76.95^\circ$$

A1 (F.T. one slip)

$$\text{or } 180^\circ - (45^\circ + 121.95^\circ) = 13.05^\circ$$

A1

(b) Area = $\frac{1}{2} \times 12 \times 10 \sin 76.95^\circ \approx 58.4 \text{ cm}^2$ (58.4 – 58.5) M1 (correct formula)

$$\text{or } \frac{1}{2} \times 12 \times 10 \sin 13.05^\circ \approx 13.4 \text{ cm}^2$$

A1 (both)

(F.T. candidate values)

6

4. (a) n th term = ar^{n-1} B1

$$\text{Let } S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} \quad (1)$$

B1 (at least 3 terms, one at each end)

$$rS_n = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n \quad (2)$$

M1 (multiplication by r and subtract)

$$(1) - (2)$$

$$S_n - rS_n = a - ar^n$$

$$S_n (1 - r) = a (1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

A1 (convincing)

- (b) (i) $ar^3 = 2, ar^6 = 54$
 $r^3 = 27$
 $r = 3$ B1 (both)
M1 (eliminate a)
A1
- (ii) $a = \frac{2}{3^3} = \frac{2}{27}$ B1 (F.T. one slip)
- $S_{10} = \frac{2}{27} \frac{(1-3^{10})}{1-3} \approx 2187.0$ M1 (correct formula,
F.T. candidate values)
A1
- (iii) $\frac{2}{27} 3^{n-1} = 125000$ B1 (F.T. candidate values)
- $\therefore 3^{n-1} = 1687500$
 $(n-1) \ln 3 = \ln 1687500$ M1 (attempt to take logs)
 $n = 1 + 13.05 = 14.05$ A1 (C.A.O.)
Least value is 15 A1 (F.T. candidate's n)

14

5. (a) $2a + d = 3$ (1)
 $a + 7d = 47$ (2)
Solve (1), (2) $d = 7, a = -2$ B1 ($a + a + d = 3$)
B1
M1 (attempt to solve)
A1 (C.A.O.)
- (b) $S_{20} = \frac{20}{2} [2 \times -2 + 19 \times 7]$ M1 (correct formula with
candidate values)
 $= 1290$ A1 (F.T. candidate values)

6

6. $\frac{5x^{\frac{4}{3}}}{4} + \frac{3x^{-2}}{-2} (+C)$
 $\frac{3}{3}$

B1, B1

$\left(\frac{15}{4} x^{\frac{4}{3}} - \frac{3}{2x^2} \right)$

2

7. (a) B $y = 0, 4 - x^2 = 0$ M1 (setting $y = 0$)

$x = \pm 2$

$B(2, 0)$

A1

A $4 - x^2 = 3x$
 $x^2 + 3x - 4 = 0$
 $x = -4, 1$
 $A(1, 3)$

M1 (equating ys)

M1 (correct attempt to solve quad)

A1

$$\begin{aligned}
 (b) \quad \text{Area} &= \int_0^1 3x dx + \int_1^2 (4 - x^2) dx && \text{M1 (use of integration to find area)} \\
 &= \left[\frac{3x^2}{2} \right]_0^1 + \left[4x - \frac{x^3}{3} \right]_1^2 && \text{M1 (addition of areas)} \\
 &= \frac{3}{2} + 8 - \frac{8}{3} - 4 + \frac{1}{3} && \text{B3 (integration)} \\
 &= \frac{19}{6} && \text{M1 (use of candidate's limits, any order)} \\
 &&& \text{A1 (C.A.O.)}
 \end{aligned}$$

12

$$\begin{aligned}
 8. \quad (a) \quad \text{Centre } (4, -2) &&& \text{B1} \\
 \text{Radius} = \sqrt{4^2 + (-2)^2} - 1 = 3 &&& \text{M1 (correct method of finding radius)} \\
 &&& \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \text{Centre is } (0, 0), \text{ radius} = a &&& \text{B1 (both)} \\
 \text{Distance between centres} = \sqrt{4^2 + 2^2} = \sqrt{20} &&& \text{B1} \\
 \text{Circles touch if}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{20} &= a + 3 && \text{M1 (F.T. distance)} \\
 a &\approx 1.47 && \text{A1 (F.T. distance)}
 \end{aligned}$$

7

$$\begin{aligned}
 9. \quad (a) \quad \frac{1}{2} 4^2 (\theta + \phi) &= 15.2 && \text{M1 (use of correct formula)} \\
 \theta + \phi &= \frac{15.2}{8} = 1.9 \quad (1) && \text{A1 (convincing)}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad 4\theta - 4\phi &= 3.2 && \text{M1 (use of correct formula)} \\
 \theta - \phi &= 0.8 \quad (2) && \text{A1} \\
 \text{Solve (1), (2)}
 \end{aligned}$$

$$\begin{aligned}
 \theta &= 1.35, \phi = 0.55 && \text{M1 (attempt to solve)} \\
 &&& \text{A1 (F.T. one slip)}
 \end{aligned}$$

6

10. (a) Let $x = a^p$, $y = a^q$
 $\log_a x = p$, $\log_a y = q$
 $xy = a^{p+q} = a^{p+q}$
 $\log_a (xy) = p + q = \log_a x + \log_a y$
 B1 (properties of $\log_a x$ and $x = a^p$)
 B1 (laws of indices)
 B1 (convincing)
- (b) $\int_1^3 \log_{10} x dx + \int_1^3 \log_{10} 10 dx$
 $\approx 0.5628 + \int_1^3 1 dx$
 $= 0.5628 + [x]^3$,
 $= 0.5628 + 3 - 1$
 $= 2.5628$
 B1 laws of logs)
 B1
 B1 (integration)
 B1

MATHEMATICS C3

1. (a) $h = 0.25$
- Integral $\approx \frac{0.25}{3} [1.7320508 + 4.2426407 + 4(2.1074644 + 3.3732634) + 2(2.6575365)]$
- ≈ 2.768
- M1 ($h = 0.25$ use of correct formula)
B1 (3 values)
B1 (2 further values)
A1 (F.T. one slip)
4
2. (a) $\theta = 30^\circ$, for example
 $\tan 2\theta = \sqrt{3} \approx 1.732$
- $2 \tan \theta = \frac{2}{3} \approx 1.15$
- ($\therefore \tan 2\theta \neq 2 \tan \theta$)
- (b) $4 (\operatorname{cosec}^2 \theta - 1) = 11 - 4 \operatorname{cosec} \theta$
- $4 \operatorname{cosec}^2 \theta + 4 \operatorname{cosec} \theta - 15 = 0$
- $(2 \operatorname{cosec} \theta - 3)(2 \operatorname{cosec} \theta + 5) = 0$
- $\operatorname{cosec} \theta = \frac{3}{2}, -\frac{5}{2}$
- $\sin \theta = \frac{2}{3}, -\frac{2}{5}$
- $\theta = 41.8^\circ, 138.2^\circ, 203.6^\circ, 336.4^\circ$
- M1 (substitution of θ in both, one correct value)
A1 (both correct and clearly unequal)
M1 (correct use of $\operatorname{cosec}^2 \theta = 1 + \cos^2 \theta$)
M1 (grouping terms and attempting to solve quad. in $\operatorname{cosec} \theta$ or $\sin \theta$ correct formula or $(a \operatorname{cosec}^2 \theta + b)(\operatorname{cosec} \theta + d)$ where $ac = \text{coefft of } \operatorname{cosec}^2 \theta$ $bd = \text{constant term}$)
A1 (CAO)
B1 ($41.5^\circ - 42^\circ$)
B1 ($203.5^\circ - 204^\circ$)
B1 ($336^\circ - 336.5^\circ$)
3. (a) $4y^3 \frac{dy}{dx} + x^3 \frac{dy}{dx} + 3x^2 y = 2x + 4$
- $4 \frac{dy}{dx} + 8 \frac{dy}{dx} + 12 = 8$
- $\frac{dy}{dx} = -\frac{4}{12}$ (o.e.)
- B1 ($4y^3 \frac{dy}{dx}$)
B1 ($x^3 \frac{dy}{dx} + 3x^2 y$)
B1 ($2x + 4$)
B1 (C.A.O.)

(b) (i) $\frac{dy}{dx} = \frac{12t^3}{6t^2}$ (o.e.)

M1 A1

(ii) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{2}{6t^2}$ (o.e.)

M1 (use of correct formula)

A1 (F.T. one slip)

8

4. (a) $\frac{2}{1+x^2} - \frac{12x}{1+x^2} - 8x = 0$

B1 $\left(\frac{2}{1+x^2}\right)$

M1 $\left(\frac{kx}{1+x^2}\right)$ A1 ($k = 12$)

$2 - 12x - 8x - 8x^3 = 0$

A1 (correct unsimplified equation, F.T. one slip)

$4x^3 + 10x - 1 = 0$

A1 (convincing)

(b) $\begin{array}{r} x \\ 0 \\ 1 \end{array} \begin{array}{r} 4x^3 + 10x - 1 \\ -1 \\ 13 \end{array}$ Change of sign indicates

M1 (attempt to find signs or values)

presence of root

A1 (correct signs, values and conclusion)

$x_0 = 0.1, x_1 = 0.0996, x_2 = 0.0996048$

B1 (x_1)

$x_3 = 0.0996047 \approx 0.099605$

B1 (x_3 , rounded or unrounded)

Check 0.0996045, 0.0996055

M1 (attempt to find signs or values)
A1 (correct)

$\begin{array}{r} x \\ 0.0996045 \\ 0.0996055 \end{array} \quad \begin{array}{r} 4x^3 + 10x - 1 \\ -0.000002 \\ 0.000008 \end{array}$

Root lies between 0.0996045 and 0.0996055 and is therefore 0.099605 to 6 decimal places

A1 (correct conclusion)

12

5. (a) $-e^{3x} \sin x + 3e^{3x} \cos x$

M1 ($f(x)e^{3x} + g(x) \cos x$)

A1 ($f(x) = -\sin x, g(x) = ke^{3x}$)

A1 ($k = 3$, correct unambiguous answer)

$$(b) \quad \frac{(3x^2 + 2)4x - (2x^2 + 1)6x}{(3x^2 + 2)^2}$$

$$= \frac{2x}{(3x^2 + 2)^2}$$

$$\text{M1} \left(\frac{(3x^2 + 2)f(x) - (2x^2 + 1)g(x)}{(3x^2 + 2)^2} \right)$$

$$\text{A1 } (f(x) = 4x, g(x) = 6x)$$

A1 (C.A.O.)

$$(c) \quad 10x \sec^2(5x^2 + 3)$$

$$\text{M1 } (\sec^2(5x + 3), \text{ allow } g(x) = 1)$$

A1 (correct unambiguous answer)

$$(d) \quad \frac{1}{2x} \times 2 = \frac{1}{x}$$

$$\text{M1 } \left(\frac{k}{2x}, \text{ allow } k = 1, 2 \right)$$

A1 (simplified answer)

$$(e) \quad \frac{3}{\sqrt{1 - (3x)^2}} \left(= \frac{3}{\sqrt{1 - 9x^2}} \right)$$

$$\text{M1 } \left(\frac{k}{\sqrt{1 - (3x)^2}} \text{ (o.e.), allow } k = 1 \right)$$

A1 ($k = 3$)

12

$$6. \quad (a) \quad 3x - 8 \leq 5$$

$$x \leq \frac{13}{3}$$

B1

$$3x - 8 \geq -5$$

M1

$$x \geq 1$$

$$1 \leq x \leq \frac{3}{13} \text{ (or } x \geq 1 \text{ and } x \leq \frac{3}{13} \text{)}$$

A1 (must indicate both conditions apply)

$$(b) \quad \text{Graphs}$$

$$\text{M1 (for } |x|, \text{ V shape through origin)}$$

A1 (translation in +ve y direction,

cusp at $(\pm 2, 1)$)

A1 (cusp at $(-2, 1)$)

A1 (correct relative positions)

7

$$7. \quad (a) \quad (i) \quad \frac{4}{7} \ln |7x + 2| - \frac{5}{6(3x + 1)^2}$$

$$\text{M1 } (k \ln(7x + 2))$$

(+C)

$$\text{A1 } \left(k = \frac{4}{7} \right)$$

$$\text{M1 } \left(\frac{k}{(3x + 1)^2} \right) \text{ A1 } \left(k = -\frac{5}{6} \text{ (o.e.)} \right)$$

(ii) $\frac{1}{2} \sin 2x (+C)$

M1 ($k \sin 2x, k = \frac{1}{2}, -\frac{1}{2}, 1, 2$)

A1 ($k = \frac{1}{2}$)

(b) $\left[2e^{\frac{x}{2}} \right]_0^4$
 $= 2e^2 - 2e^0$
 ~ 12.8

M1 ($ke^{\frac{x}{2}}, k = 2, \frac{1}{2}, 1$)

A1 ($k = 2$)

M1 ($ke^2 - ke^0$, allowable ks)

A1 (C.A.O. at least 3 sig. figs)

10

8. (a) Let $y = 3x^2 + 4$
 $x^2 = \frac{y-4}{3}$
 $x = \pm \sqrt{\frac{y-4}{3}}$

M1 ($x^2 = f(y)$)

Choose + $\because$ domain of f is $x \geq 0$

A1

$x = \sqrt{\frac{y-4}{3}}$

A1 (F.t. one slip)

$f^{-1}(x) = \sqrt{\frac{x-4}{3}}$

A1 (F.T. one slip)

Domain is $x \geq 4$, range is $f^{-1}(x) \geq 0$ (o.e.)

B1, B1

Domain is $x \geq 4$, range is $f^{-1}(x) > 0$

(b) Graphs

M1 (full or half parabola passing through (0,b))

A1 (r.h. branch and minimum at (0,4))

A1 F.T. b, full or half parabola))

9

9. (a) Domain is $(2, \infty)$

B1

(b) ($f(g(x)) = 5$)

$e^{\ln(x^2-4)} = 5$

$= 5$

M1 (correct order)

$x^2 - 4 = 5$

or

$\ln(x^2 - 4) = \ln 5$

A1 (either)

$x^2 = 9$

A1

$x = 3$ (-3 not in domain)

A1 (with reason)

5

MATHEMATICS FP1

1. $\text{Mod} = \sqrt{3+1} = 2$ B1
- $\text{Arg} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} \quad (30^\circ)$ B1
- $\sqrt{3} + i = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$ B1
- $\text{Arg of } (\sqrt{3} + i)^n = \frac{n\pi}{6}$ M1A1
- Power is real when
- $\frac{n\pi}{6}$ is an integer multiple of π A1
- Smallest $n = 6$. A1
-
2. $\text{Det} = 1.2 + 2\lambda + \lambda(3\lambda - 4)$ M1m1A1
- $= 3\lambda^2 - 2\lambda + 2$ A1
- EITHER $= 3((\lambda - 1/3)^2 + 5/9)$ M1
- > 0 for all λ A1
- OR $'b^2 - 4ac' = -20$ M1
- So determinant not equal to zero for any real λ . A1
-
3. $f(x+h) - f(x) = \frac{1}{1-(x+h)^2} - \frac{1}{1-x^2}$ M1A1
- $= \frac{1-x^2 - [1-(x+h)^2]}{[1-(x+h)^2](1-x^2)}$ m1
- $= \frac{h(2x+h)}{[1-(x+h)^2](1-x^2)}$ A1
- $f'(x) = \lim_{h \rightarrow 0} \frac{(2x+h)}{[1-(x+h)^2](1-x^2)}$ M1
- $= \frac{2x}{(1-x^2)^2}$ A1
-
4. $\frac{11+7i}{1+i} = \frac{(11+7i)(1-i)}{(1+i)(1-i)}$ M1A1
- $= \frac{18-4i}{2} = 9-2i$ A1
- $2(x+iy) + (x-iy) = 9-2i$ M1
- Equating real and imaginary parts m1
- $x = 3$ and $y = -2$ A1

5. (a) $\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$\mathbf{T}_2 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ M1

$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ A2

Note: Multiplying the wrong way gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Award M1A1}$$

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{M1}$$

giving $-y - 2 = x$ and $x + 1 = y$

Fixed point is $(-3/2, -1/2)$.

FT from wrong answer to (a) – the incorrect matrix above leads to $(-1/2, 3/2)$.

A1
M1A1

6. (a) Assume the proposition is true for $n = k$, that is

$$\sum_{r=1}^k (2r+1) = (k+1)^2 \quad \text{B1}$$

Consider

$$\begin{aligned} \sum_{r=1}^{k+1} (2r+1) &= (k+1)^2 + 2k + 3 \\ &= (k+2)^2 \end{aligned} \quad \text{M1A1} \quad \text{A1}$$

So, if the proposition is true for $n = k$, it is also true for $n = k + 1$.

A1

(b) Since $3 \neq 4$, P is false for $n = 1$ is therefore false.

B2

7. (a) Using reduction to echelon form,
- $$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & \lambda - 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu - 4 \end{bmatrix}$$
- M1A1A1
- $$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & \lambda - 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu - 10 \end{bmatrix}$$
- A1
- The solution will not be unique if $\lambda = 3$. A1
- (b) The equations are consistent if $\mu = 10$. B1
Put $z = \alpha$ M1
 $y = \alpha - 6$ A1
 $x = 16 - 4\alpha$ M1A1
8. (a) Let the roots be $\alpha - 1, \alpha, \alpha + 1$. M1
Then $\alpha(\alpha - 1) + \alpha(\alpha + 1) + (\alpha - 1)(\alpha + 1) = 47$ m1
 $\alpha^2 = 16$ A1
 $\alpha = -4$ A1
The roots are $-3, -4$ and -5 . A1
- (b) $p = 12, q = 60$ B1B1
9. (a) $\ln f(x) = \frac{1}{x} \ln(x)$ M1
 $\frac{f'(x)}{f(x)} = -\frac{1}{x^2} \ln(x) + \frac{1}{x^2}$ m1A1
 $= \frac{1}{x^2} (1 - \ln(x))$ A1
At the stationary point,
 $\ln(x) = 1$ M1
so $x = e$ (2.72) and $y = e^{\frac{1}{e}}$ (1.44) A1A1
- (b) We now need to determine its nature.
We see from above that
For $x < e$, $f'(x) > 0$ and for $x > e$, $f'(x) < 0$ M1
Showing it to be a maximum. A1

10.

$$w = \frac{z+3}{z+1}$$

$$wz + w = z + 3 \quad \text{M1}$$

$$z(w-1) = 3-w \quad \text{A1}$$

$$z = \frac{3-w}{w-1} \quad \text{A1}$$

Since $|z| = 1$, it follows that

$$|3-w| = |w-1| \quad \text{M1}$$

$$\sqrt{(3-u)^2 + v^2} = \sqrt{(u-1)^2 + v^2} \quad \text{A1}$$

$$9 - 6u + u^2 + v^2 = u^2 - 2u + 1 + v^2 \quad \text{A1}$$

leading to $u = 2$. A1

Straight line parallel to the v -axis (passing through (2,0)). B1

ALTERNATIVE METHOD

$$u + iv = \frac{x+3+iy}{x+1+iy} \cdot \frac{x+1-iy}{x+1-iy} \quad \text{M1}$$

$$= \frac{(x+1)(x+3) + y^2 + iy(x+1-x-3)}{(x+1)^2 + y^2} \quad \text{A1}$$

Equating real and imaginary parts, M1

$$u = \frac{x^2 + y^2 + 4x + 3}{x^2 + y^2 + 2x + 1} \quad \text{A1}$$

$$v = \frac{-2y}{x^2 + y^2 + 2x + 1} \quad \text{A1}$$

Putting $x^2 + y^2 = 1$, M1

$$u = 2 \quad \text{A1}$$

which is a straight line parallel to the v -axis (passing through (2,0)). B1

MATHEMATICS M1

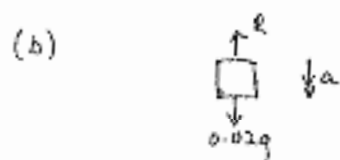
(a) (i) Use of $v^2 = u^2 + 2as$ with $u = 0$, $a = (\pm)9.8$, $s = (\pm)160$ M1
 $v^2 = 0 + 2 \times 9.8 \times 160$ A1

$v = \underline{56 \text{ ms}^{-1}}$ A1

(ii) Use of $s = ut + \frac{1}{2}at^2$ with $s = (\pm)160$, $u = 0$, $a = (\pm)9.8$ M1

$(-160) = \frac{1}{2} \times (-)9.8 t^2$ ft v A1/

$t = \underline{\frac{40}{7} \text{ s}} = 5\frac{5}{7} \approx 5.71$ A1



use of 0.2 for mass make an error

(i) Use of N2L dim. correct M1

$0.02g - 0.046 = 0.02a$ A1

$a = \underline{5 \text{ ms}^{-2}}$ A1

(ii) Use of $s = ut + \frac{1}{2}at^2$ with $u = 0$, $a = (\pm)5$, $t = 4$ M1

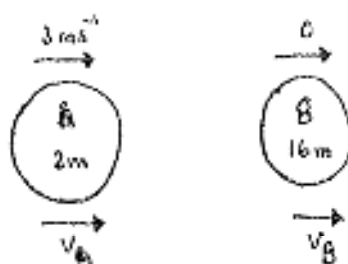
$s = \frac{1}{2} \times 5 \times 4^2$ ft a A1/

$= 40$

$\therefore \text{Height above ground} = 160 - 40$

$= \underline{120 \text{ m}}$ A1/

2(a)



Conservation of momentum

allow $v_A = 0$ M1

$$2 \times 3(m) + 16 \times 0(m) = 2v_A(m) + 16v_B(m)$$

A1

$$v_A + 8v_B = 3$$

Restitution

allow $v_B = 0$ M1

$$v_B - v_A = -\frac{1}{2}(0 - 3)$$

A1

$$-v_A + v_B = \frac{3}{2}$$

Adding

$$v_B = \frac{9}{2}$$

dep on both M1

$$v_B = \frac{1}{2} \text{ ms}^{-1}$$

ft 1 slip A1

$$v_A = v_B - \frac{3}{2}$$

$$= -1 \text{ ms}^{-1}$$

ft 1 slip A1

Allow v_A going to left.
consistent equations

(b) Impulse $I = 2m(-1 - 3)$

M1

$$= -8m \text{ NS} = \text{kg m s}^{-1}$$

allow no =

A1 B1

in the direction ^{opposite to} the original motion of A

A1

3 (a)

$$\tau = \frac{600}{20}$$

$$= \underline{30}$$

81

(b) using $s = ut + \frac{1}{2}at^2$ with $u = 15$, $s = 600$, $t = 30$ M1

$$600 = 15 \times 30 + \frac{1}{2}a \times 30^2$$

A1

$$a = \underline{\frac{1}{3} \text{ ms}^{-2}}$$

accept 0.3, 0.33 etc

A1

(c) using $s = \frac{1}{2}(u+v)t$ with $s = 600$, $u = 15$, $t = 30$ M1

$$600 = \frac{1}{2}(15+v) \times 30$$

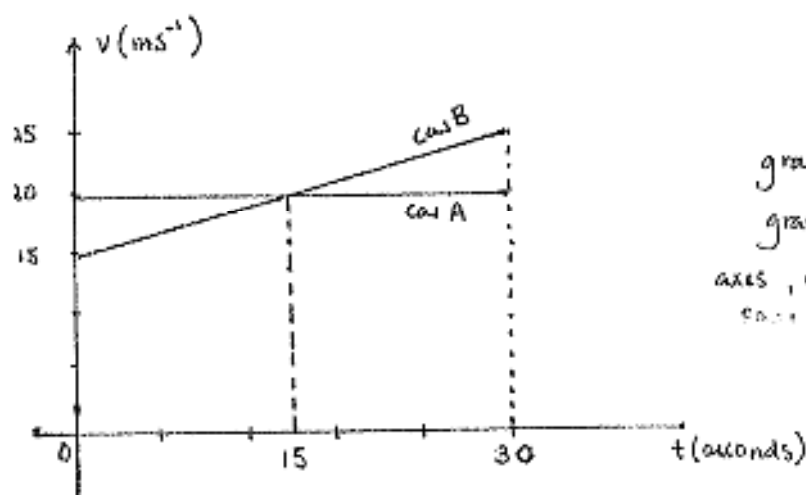
A1

$$v = \underline{25 \text{ ms}^{-1}}$$

can A1

PH of 0.3 or 0.33 used

(d)

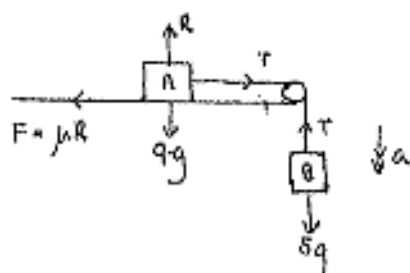


graph A 81
graph B 81
axes, scales etc 81
same graph

required time when A and B^A have the same speed is 15 s

it's v 81

4 (a)



M2L to B $T = 5g = 49$ B1

M2L to A $T = F = 49$ B1

Resolve $\uparrow$ for A $R = 9g = 88.2$ B1

System in equilibrium

$$F \leq \mu R$$

$$\mu \geq \frac{5g}{9g} = \frac{5}{9}$$

for $F = \mu R$ and $a = 0$ M1

correctly A1

(b)

$$R = 9g = 88.2$$

$$F = \mu R$$

$$= 0.5 \times 9 \times 9.8$$

$$= 4.5g = 44.1$$

B1

M2L to B

$$5g - T = 5a$$

M1 A1

M2L to A

$$T - F = 9a$$

M1 A1

$$T - 4.5g = 9a$$

Adding

$$5g - 4.5g = 14a$$

dep on both M1s. M1

$$a = \frac{0.5 \times 9.8}{14}$$

$$= \underline{0.35 \text{ ms}^{-2}}$$

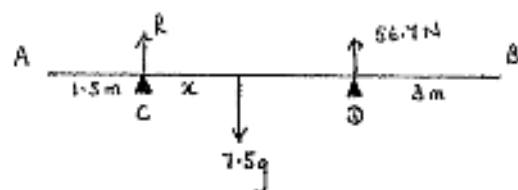
A1

$$T = 9 \times 0.35 + 4.5 \times 9.8$$

$$= \underline{47.25 \text{ N}}$$

dep. only on one M1

5.



(a) Moment about C

$$7.5g \times x = 56.7 \times (5 - 1.5)$$

$$x = \underline{2.7 \text{ m.}}$$

dim. correct

M1

A1 G1

cao A1

(b) Resolve $\uparrow$

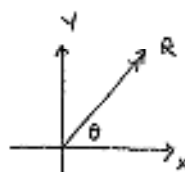
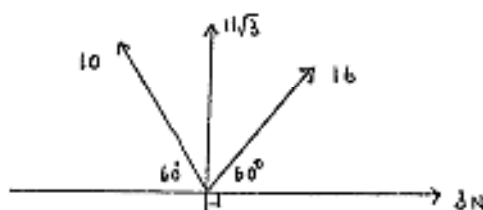
$$R + 56.7 = 7.5g$$

$$R = \underline{16.8 \text{ N}}$$

M1

A1

6.

Resolve $\rightarrow$

$$x = 3 + 16 \cos 60^\circ - 10 \cos 60^\circ$$

$$= \underline{6 \text{ N}}$$

resolution

M1

A1 all for
first 2 = 1
A1Resolve $\uparrow$

$$y = 11\sqrt{3} + 10 \sin 60^\circ + 16 \sin 60^\circ$$

$$= \underline{24\sqrt{3} \text{ N}} \approx 41.569 \text{ N}$$

resolution

M1

A1 all for
(incl.)
A1

$$R = \sqrt{6^2 + (24\sqrt{3})^2}$$

$$= \underline{42 \text{ N}}$$

M1

ft x, y
penalise PA.

A1✓

$$\tan \theta = \frac{24\sqrt{3}}{6}$$

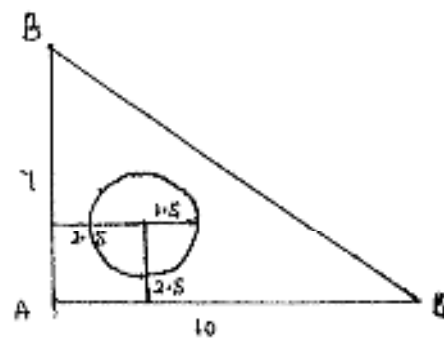
M1

$$\theta = \underline{81.8^\circ}$$

ft x, y.

A1✓

7.



			from AB	from AC	
(a)	ΔABC	35	$\frac{10}{3}$	$\frac{7}{3}$	Area
	Circle	$\pi \left(\frac{3}{2}\right)^2 = 7.069$	$\frac{5}{2}$	$\frac{5}{2}$	Area
	Lamina	$35 - \frac{9}{4}\pi = 27.92$	x	y	

$$(i) \quad 35 \times \frac{10}{3} = \frac{9}{4}\pi \times \frac{5}{2} + (35 - \frac{9}{4}\pi)x$$

$$x = \underline{3.54}$$

$$(ii) \quad 35 \times \frac{7}{3} = \frac{9}{4}\pi \times \frac{5}{2} + (35 - \frac{9}{4}\pi)y$$

$$y = \underline{2.29}$$

(b) Required angle $\theta = \tan^{-1}\left(\frac{2.29}{3.54}\right)$

$$= \underline{32.9^\circ}$$

M1 – ASSESSMENT OBJECTIVES

Q.No.	AO1	AO2	AO3	AO4	AO5	Total
1	2	2	6	2		12
2	3	2	4	2		11
3	2	3	4	2		11
4	3	3	4	1	2	13
5	1	1	2	2		6
6	2	2	2	1	3	10
7	3	3	3	2	1	12
TOTAL	16	16	25	12	6	75

MATHEMATICS S1

1. (a) $P(\text{Score at least 3 on one die}) = \frac{4}{6}$ B1

$P(\text{Score at least 3 on both dice}) = \left(\frac{4}{6}\right)^2 = \frac{4}{9}$ M1A1

[Award B1 for the 6×6 array of possible scores, M1 for counting the number of these satisfying the required condition and A1 for the correct answer]

(b) Possibilities are 6,3;5,2;4,1 and reverse, ie 6 possibilities M1A1

$$\text{Prob} = \frac{6}{36} \left(\frac{1}{6}\right) (\text{cao})$$
 A1

2. (a) $P(A \cup B) = P(A) + P(B)$ gives $P(B) = 0.2$ M1A1

(b) $0.5P(B) = 0.5 + P(B) - 0.7$ M1A1
 $0.5P(B) = 0.2$ gives $P(B) = 0.4$ M1A1

(c) $0.5 \times 0.3 = 0.5 + P(B) - 0.7$ M1A1
 $P(B) = 0.35$ A1

3. (a) 2. B1

(b) (i) $\text{Prob} = e^{-4} \cdot \frac{4^3}{3!} = 0.195$ 1A1

[Accept $0.7619 - 0.5665$ or $0.4335 - 0.2381$ from tables, M1 for at least 1 correct value]

(ii) $\text{Prob} = 0.9084 - 0.1107$ or $0.8893 - 0.0916 = 0.7977$ (cao) B1B1B1

(c) $E(C) = 5 + 4 \times 4 = 21$ M1A1
 $SD = \sqrt{16 \times \text{Var}(X)} = 8$ M1A1

[Award M1 for 64]

4. (a) $\text{Prob} = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$ or $\frac{\binom{5}{3}}{\binom{8}{3}} = \frac{5}{28}$ M1A1

$$(b) \quad P(3 \text{ blue}) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \text{ or } \frac{\binom{3}{3}}{\binom{8}{3}} \quad (= \frac{1}{56}) \quad \text{B1}$$

$$P(2 \text{ blue}) = 3 \times \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} \text{ or } \frac{\binom{3}{2} \times \binom{5}{1}}{\binom{8}{3}} \quad (= \frac{15}{56}) \quad \text{M1A1}$$

$$\text{Reqd prob} = \frac{16}{56} \quad (\frac{2}{7}) \quad \text{B1}$$

5. $np = 20$; $np(1-p) = 16$ M1A1A1
Valid attempt to solve equations (FT) M1
 $p = 0.2$ B1
 $n = 100$ B1

6. (a) $P(4) = \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{4}$ M1A1
 $= \frac{5}{24}$ A1

(b) $P(\text{cube} | 4) = \frac{1/12}{5/24} = \frac{2}{5} \text{ (cao)}$ B1B1B1

7. (a) Number of defective glasses X is $B(24, 0.05)$ (si) B1
 $P(X=2) = \binom{24}{2} \times 0.05^2 \times 0.95^{22} = 0.223$ M1A1

(b) $P(3 \leq X \leq 5) = 0.9622 - 0.5405 \text{ or } 0.4595 - 0.0378 = 0.422 \text{ (cao)}$ B1B1B1

(c) Number of defective glasses is now $B(120, 0.05) \approx \text{Poi}(6)$ B1
 $P(Y < 8) = 0.744 \text{ (or } 1 - 0.256)$ M1A1

8. (a) $[0, 0.7]$ [Accept (0.0.7)] B1B1
[Award B1 for $\{0, 0.1, \dots, 0.7\}$ or similar]

(b) (i) $E(X) = 0.1 + 0.4 + 3\theta + 4(0.7 - \theta) = 3.3 - \theta$ M1A1
 $3.3 - \theta = 3$ gives $\theta = 0.3$ M1A1

(ii) $E(X^3) = 0.1 \times 1 + 0.2 \times 8 + 0.3 \times 27 + 0.4 \times 64$ M1A1
 $= 35.4$ A1

9.	(a)	(i)	$\int_1^4 kx^2 dx = \frac{k}{3} [x^3]_1^4 = 21k$	M1A1
			Int = 1 gives $k = \frac{1}{21}$	A1
		(ii)	$E(X) = \frac{1}{21} \int_1^4 x \cdot x^2 dx$ (Limits required, here or later)	M1
			$= \frac{1}{21} \cdot \frac{1}{4} [x^4]_1^4$	B1
			$= \frac{85}{28} \text{ (3.04)}$	A1
	(b)	(i)	$F(x) = \frac{1}{21} \int_1^x y^2 dy$	M1
			$= \frac{1}{63} [y^3]_1^x$	A1
			$= \frac{1}{63} (x^3 - 1)$	A1
		(ii)	Prob = $F(3) - F(2)$	M1
			$= \frac{19}{63} \text{ (0.302)}$	A1
		(iii)	The median m satisfies	
			$\frac{1}{63} (m^3 - 1) = 0.5$	M1
			$m^3 = 32.5$	A1
			$m = 3.19 \text{ (cao)}$	A1

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