

983/01

MATHEMATICS S1

Statistics

P.M. WEDNESDAY, 18 January 2006

(1 $\frac{1}{2}$ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Two unbiased cubical dice are thrown simultaneously. Calculate the probability that
 - (a) the score on each die is at least 3, [3]
 - (b) the scores on the two dice differ by 3. [3]

2. The events A and B are such that $P(A) = 0.5$ and $P(A \cup B) = 0.7$. Determine the value of $P(B)$ in each of the cases when
 - (a) A and B are mutually exclusive, [2]
 - (b) A and B are independent, [4]
 - (c) $P(B|A) = 0.3$. [3]

3. The number of machine breakdowns, X , occurring in a certain factory in a randomly chosen week may be assumed to have a Poisson distribution with mean 4.
 - (a) Write down the standard deviation of X . [1]
 - (b) Find the probability that the number of machine breakdowns in a randomly chosen week is
 - (i) exactly 3,
 - (ii) between 2 and 6 (both inclusive). [5]
 - (c) The cost C (in appropriate monetary units) of repairing these machines is given by

$$C = 5 + 4X.$$
 Find the mean and standard deviation of C . [4]

4. A bag contains 5 red balls and 3 blue balls. A random sample of 3 balls is selected from the bag, without replacement. Calculate the probability that
 - (a) all the selected balls are red, [2]
 - (b) more blue balls are selected than red balls. [4]

5. The random variable X has the binomial distribution $B(n, p)$. The mean and standard deviation of X are 20 and 4 respectively. Find the values of n and p . [6]

6. Jim has a fair cubical die with the six faces numbered 1, 2, 3, 4, 5, 6 respectively and a fair tetrahedral die with the four faces numbered 1, 2, 3, 4 respectively. He tosses a fair coin. If it falls 'heads', he throws the cubical die. If it falls 'tails', he throws the tetrahedral die.
 - (a) Calculate the probability that he obtains a '4'. [3]
 - (b) Given that he obtains a '4', find the probability that he threw the cubical die. [3]

7. Wine glasses are mass produced. There is a probability of 0.05 that a randomly selected glass is defective, independently of all other glasses.
- (a) **Without using tables**, find the probability that a set of 24 glasses contains exactly 2 defective glasses. [3]
- (b) **Using tables**, find the probability that a set of 50 glasses contains between 3 and 5 (both inclusive) defective glasses. [3]
- (c) Use a Poisson approximation to find the probability that a set of 120 glasses contains fewer than 8 defective glasses. [3]
8. The following table gives the probability distribution of the discrete random variable X , where θ is a constant.

x	1	2	3	4
$P(X = x)$	0.1	0.2	θ	$0.7 - \theta$

- (a) State the range of possible values of θ . [2]
- (b) Given that $E(X) = 3$,
- (i) find the value of θ ,
- (ii) evaluate $E(X^3)$. [7]
9. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= kx^2 & \text{for } 1 \leq x \leq 4, \\ f(x) &= 0 & \text{otherwise,} \end{aligned}$$

where k is a constant.

- (a) (i) Show that $k = \frac{1}{21}$.
- (ii) Evaluate $E(X)$. [6]
- (b) (i) Obtain an expression for $F(x)$, valid for $1 \leq x \leq 4$, where F denotes the cumulative distribution function of X .
- (ii) Evaluate $P(2 \leq X \leq 3)$.
- (iii) Find the median of X . [8]