

973/01

MATHEMATICS C1

Pure Mathematics

P.M. WEDNESDAY, 10 January 2007

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A, B, C, D have coordinates $(-5, 0), (0, 5), (3, 4), (4, -3)$, respectively.

(a) Show that AC is perpendicular to BD . [4]

(b) Show that AD is parallel to BC . [3]

(c) Show that the equation of AC is

$$x - 2y + 5 = 0$$

and find the equation of BD .

[3]

(d) The lines AC and BD intersect at E .

(i) Show that the coordinates of E are $(1, 3)$. [2]

(ii) Find the length of AE . [2]

2. Simplify **each** of the following expressions, expressing your answers in surd form.

(a) $2\sqrt{32} + 3\sqrt{8} - \sqrt{18}$ [3]

(b) $\frac{6 + \sqrt{30}}{6 - \sqrt{30}}$ [4]

3. When $9x^3 + 6x^2 - 5x + p$ is divided by $x - 1$, the remainder is 8.

(a) Show that $p = -2$. [2]

(b) Factorise $9x^3 + 6x^2 - 5x - 2$. [5]

4. (a) Expand $(a + b)^4$, simplifying your coefficients as much as possible. [2]

(b) Solve $(2 + x)^4 = 14 + 33x + 25x^2 + 8x^3 + x^4$. [4]

5. (a) Given that $y = 2x^2 - 5x + 3$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Find the equation of the normal to the curve $y = 2x^2 - 5x + 3$ at the point $(2, 1)$. [3]

6. Differentiate **each** of the following with respect to x .

(a) $2x^5 + \frac{24}{x^2} - 3\sqrt{x}$ [3]

(b) $x^2(3x + 1)$ [2]

7. Given that the equation

$$kx^2 - 4x + (k - 3) = 0$$

has real roots, show that

$$k^2 - 3k - 4 \leq 0.$$

Find the range of values of k satisfying this inequality. [7]

8. (a) Express $x^2 + 4x + 9$ in the form $(x + a)^2 + b$, where the values of a and b are to be determined.
Deduce the maximum value of

$$\frac{1}{x^2 + 4x + 9}.$$
 [4]

- (b) Show that the line $y = x + 2$ touches the curve $y = x^2 - 5x + 11$, and find the coordinates of the point of contact. [4]

9. The curve C has equation

$$y = 4x^3 - 12x + 3.$$

- (a) Find the coordinates of the stationary points of C and determine the nature of each of these points. [7]
- (b) Sketch C , indicating the coordinates of the stationary points. [3]
- (c) Given that $f(x) = 4x^3 - 12x + 3$, sketch the curve $y = f(x - 1)$, indicating the coordinates of **each** of the stationary points. [3]