General Certificate of Education
Advanced Subsidiary/Advanced

983/01

MATHEMATICS S1
Statistics

P.M. THURSDAY, 17 January 2008
(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The two events A, B are such that

$$P(A) = 0{\cdot}3,\ P(B) = 0{\cdot}1,\ P(A \cup B) = 0{\cdot}35.$$

(a) Evaluate $P(A \cap B)$. [2]

(b) Determine whether or not A and B are independent. [3]

(c) Evaluate $P(A \mid B')$. [4]

2. A group of 10 children, 6 girls and 4 boys, is on a school visit to the theatre. The teacher is asked to select 3 of these children to meet members of the cast after the show and she decides to do the selection at random.

(a) Calculate the probability that she selects

(i) 3 girls, [2]

(ii) more boys than girls. [4]

(b) Ann is one of the children on the visit. Calculate the probability that she is one of the selected children. [2]

3. When Alan types a report, the number of errors on each page has a Poisson distribution with mean 0·95, independently of all other pages.

(a) Without the use of tables, find the probability that a randomly selected page contains

(i) no errors,

(ii) either 3 or 4 errors. [5]

(b) Alan types a 4-page report. Calculate the probability that

(i) there are no errors anywhere in the report,

(ii) the first error occurs on the third page. [5]

4. The random variable X has the binomial distribution $B(10, 0{\cdot}3)$. Given that $Y = 3X + 4$, evaluate

(a) $E(Y)$, [4]

(b) $\mathrm{Var}(Y)$, [2]

(c) $P(Y = 16)$. [3]

5. A factory has three machines making paper clips. Machine A produces 40% of the total output, Machine B produces 35% of the total output and Machine C produces 25% of the total output. It is known that 2% of the paper clips produced by Machine A are defective, 2·5% produced by Machine B are defective and 0·5% produced by Machine C are defective.
A paper clip is selected at random from the total output.

(a) Calculate the probability that it is defective. [3]

(b) Given that it is defective, find the probability that it was produced by Machine A. [3]

6. The discrete random variable X has the following probability distribution.

x	1	2	3
$P(X = x)$	θ	2θ	$1 - 3\theta$

(a) State the range of possible values of the constant θ. [2]

(b) Given that $E(X) = 2{\cdot}2$,

(i) show that $\theta = 0{\cdot}2$,

(ii) calculate the standard deviation of X,

(iii) evaluate $E\left(\frac{1}{X}\right)$. [10]

7. On a farm, chickens are bred from eggs under strictly controlled conditions.

(a) The probability that an egg will produce a female chick is 0·3. When 20 eggs are kept under the controlled conditions, find the probability that the number of female chicks produced will be

(i) exactly 8,

(ii) more than 5. [5]

(b) The probability that an egg will fail to hatch is 0·01. When 1000 eggs are kept under the controlled conditions, use a Poisson approximation to find the probability that the number of eggs failing to hatch will be less than 9. [3]

TURN OVER.

8. The continuous random variable X has probability density function f given by

$$f(x) = 4 - 2x, \qquad \text{for } 1 \leqslant x \leqslant 2,$$

$$f(x) = 0, \qquad \text{otherwise.}$$

(a) Evaluate $E(X)$. [4]

(b) Show that, for $1 \leqslant x \leqslant 2$,

$$F(x) = 4x - x^2 - 3$$

where F denotes the cumulative distribution function of X. [3]

(c) Evaluate $P(X > 1{\cdot}2)$. [3]

(d) Calculate the median of X, giving your answer correct to two decimal places. [3]