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GENERAL CERTIFICATE OF EDUCATION
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# MARKING SCHEME

# MATHEMATICS
# AS/Advanced

## JANUARY 2009

## INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2009 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

# Mathematics C1 January 2009

# Solutions and Mark Scheme

# Final Version

| | | | |
|---|---|---|---|
| **1.** | (*a*) | Gradient of $BC = \frac{\text{increase in } y}{\text{increase in } x}$ | M1 |
| | | Gradient of $BC = {}^1/_4$ (or equivalent) | A1 |
| | | A correct method for finding the equation of $BC(AD)$ using candidate's gradient for $BC$ | M1 |
| | | Equation of $BC$ : $y - 4 = {}^1/_4\,(x - 5)$ (or equivalent) (f.t. candidate's gradient for $BC$) | A1 |
| | | Equation of $BC$ : $x - 4y + 11 = 0$ (convincing) | A1 |
| | | Use of $m_{AB} \times m_{CD} = -1$ | M1 |
| | | Equation of $AD$ : $y - (-1) = -4(x - 2)$ (or equivalent) (f.t. candidate's gradient of $BC$) | A1 |
| | | **Special case:** | |
| | | Verification of equation of $BC$ by substituting coordinates of **both** $B$ and $C$ into the given equation | M1 |
| | | Making an appropriate statement | A1 |
| | (*b*) | An attempt to solve equations of $BC$ and $AD$ simultaneously | M1 |
| | | $x = 1$, $y = 3$ (convincing) (c.a.o.) | A1 |
| | | **Special case** | |
| | | Substituting (1, 3) in equations of **both** $BC$ and $AD$ | M1 |
| | | Convincing argument that coordinates of $D$ are (1, 3) | A1 |
| | (*c*) | A correct method for finding the length of $CD$ | M1 |
| | | $CD = \sqrt{17}$ | A1 |
| | (*d*) | A correct method for finding $E$ | M1 |
| | | $E(0, 7)$ | A1 |

**2.** (*a*) $\dfrac{10\sqrt{3} - 1}{4 - \sqrt{3}} = \dfrac{(10\sqrt{3} - 1)(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$ M1

Numerator: $40\sqrt{3} + 10 \times 3 - 4 - \sqrt{3}$ A1

Denominator: $16 - 3$ A1

$\dfrac{10\sqrt{3} - 1}{4 - \sqrt{3}} = \dfrac{39\sqrt{3} + 26}{13} = 3\sqrt{3} + 2$ (c.a.o.) A1

**Special case**

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 - \sqrt{3}$

(*b*) $(2 + \sqrt{5})(5 - \sqrt{20}) = 10 - 2\sqrt{20} + 5\sqrt{5} - \sqrt{5} \times \sqrt{20}$

(4 terms, at least 3 correct) M1

$\sqrt{20} = 2\sqrt{5}$ B1

$\sqrt{5} \times \sqrt{20} = 10$ B1

$(2 + \sqrt{5})(5 - \sqrt{20}) = \sqrt{5}$ (c.a.o.) A1

**Alternative Mark Scheme**

$(2 + \sqrt{5})(5 - \sqrt{20}) = (2 + \sqrt{5})(5 - 2\sqrt{5})$ B1

$(2 + \sqrt{5})(5 - 2\sqrt{5}) = 10 - 4\sqrt{5} + 5\sqrt{5} - \sqrt{5} \times 2\sqrt{5}$

(4 terms, at least 3 correct) M1

$\sqrt{5} \times 2\sqrt{5} = 10$ B1

$(2 + \sqrt{5})(5 - \sqrt{20}) = \sqrt{5}$ (c.a.o.) A1

**3.** (*a*) $\dfrac{dy}{dx} = 2x - 9$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 6$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Gradient of tangent at $P = 3$ (c.a.o.) A1

Equation of tangent at $P$: $y - (-5) = 3(x - 6)$ (or equivalent)

(f.t. candidate's value for $\dfrac{dy}{dx}$ provided both M1 and m1 awarded) A1

(*b*) Use of gradient of tangent at $Q \times {}^{1}/_{7} = -1$ M1

Equating candidate's expression for $\dfrac{dy}{dx}$ and candidate's value for gradient of tangent at $Q$ m1

$2x - 9 = -7 \Rightarrow x = 1$ (f.t. candidate's expression for $\dfrac{dy}{dx}$) A1

**4.** $a = 3$ B1

$b = -2$ B1

$c = 5$ B1

A positive quadratic graph M1

Minimum point $(-b, c)$ A1

**5.** An expression for $b^2 - 4ac$, with at least two of $a$, $b$, $c$ correct M1
$b^2 - 4ac = 8^2 - 4 \times (3k - 2) \times k$ A1
Putting $b^2 - 4ac < 0$ m1
$3k^2 - 2k - 16 > 0$ (convincing) A1
Finding critical points $k = -2$, $k = {}^8/_3$ B1
A statement (mathematical or otherwise) to the effect that
$k < -2$ or ${}^8/_3 < k$ (or equivalent) (f.t. candidate's critical points) B2
Deduct 1 mark for each of the following errors
the use of non-strict inequalities
the use of the word 'and' instead of the word 'or'

**6.** (*a*) $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (–1 for each error)
(–1 for any subsequent 'simplification') B2

(*b*) An expression containing $k \times (1/4)^2 \times (2x)^3$, where $k$ is an integer $\neq 1$ and is either the candidate's coefficient for the $a^2b^3$ term in (*a*) or is derived from first principles M1
Coefficient of $x^3 = 5$ (c.a.o.) A1

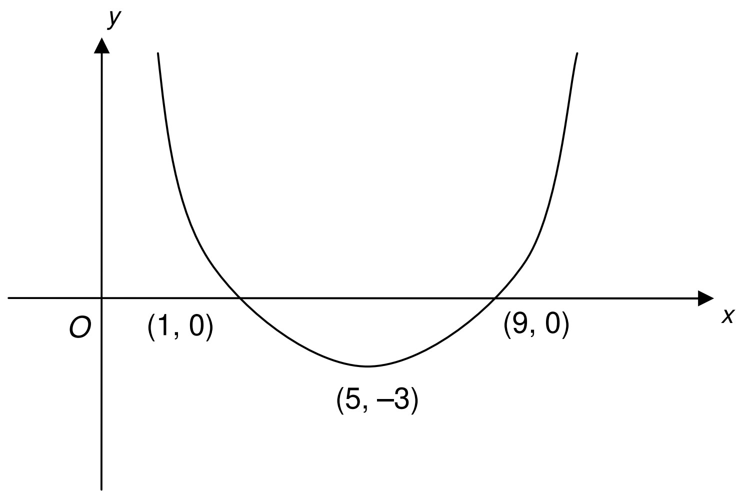
**7.** (*a*) An attempt to calculate $3^3 - 17$ M1
Remainder = 10 A1

(*b*) Attempting to find $f(r) = 0$ for some value of $r$ M1
$f(2) = 0 \Rightarrow x - 2$ is a factor A1
$f(x) = (x - 2)(6x^2 + ax + b)$ with one of $a$, $b$ correct M1
$f(x) = (x - 2)(6x^2 + 5x - 4)$ A1
$f(x) = (x - 2)(3x + 4)(2x - 1)$ (f.t. only $6x^2 - 5x - 4$ in above line) A1
Roots are $x = 2, -{}^4/_3, {}^1/_2$ (f.t. for factors $3x \pm 4$, $2x \pm 1$) A1
**Special case**
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1

**8.** (*a*) $y + \delta y = 7(x + \delta x)^2 + 5(x + \delta x) - 2$ B1
Subtracting $y$ from above to find $\delta y$ M1
$\delta y = 14x\delta x + 7(\delta x)^2 + 5\delta x$ A1
Dividing by $\delta x$ and letting $\delta x \to 0$ M1
$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x + 5$ (c.a.o.) A1

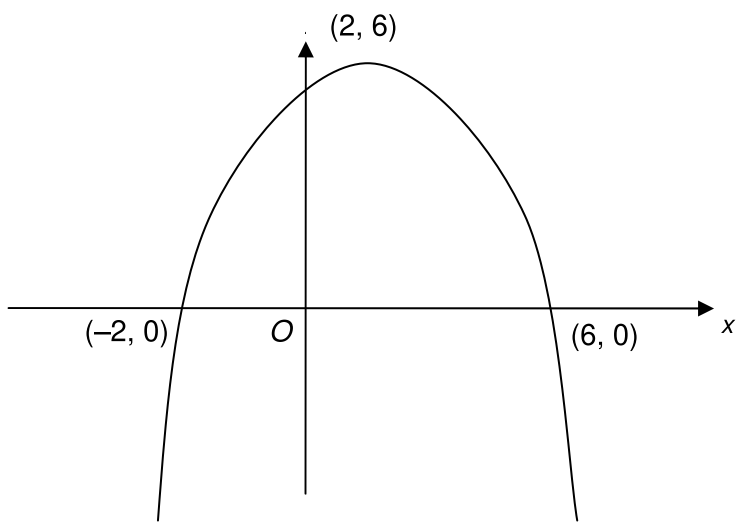
(*b*) Required derivative $= 2 \times (-3) \times x^{-4} + 5 \times ({}^2/_3) \times x^{-1/3}$ B1, B1

**9.** (*a*)




| | |
|---|---|
| Concave up curve and $y$-coordinate of minimum = –3 | B1 |
| $x$-coordinate of minimum = 5 | B1 |
| Both points of intersection with $x$-axis | B1 |

(*b*)




| | |
|---|---|
| Concave down curve and $x$-coordinate of maximum = 2 | B1 |
| $y$-coordinate of maximum = 6 | B1 |
| Both points of intersection with $x$-axis | B1 |

**10.** (*a*) $\frac{dy}{dx} = 3x^2 + 6x - 9$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = -3, 1$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are (–3, 14) and (1, –18) (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

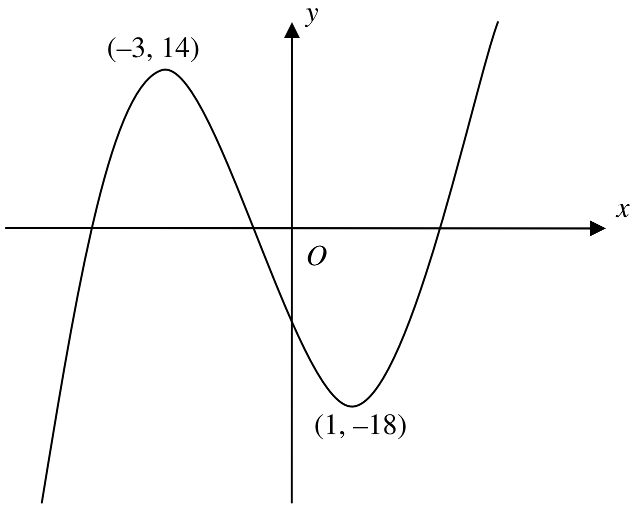
**either** (–3, 14) is a maximum point

**or** (1, –18) is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)




Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) A statement identifying the number of roots as the number of times the curve crosses the *x*-axis (any curve) M1

Correct interpretation of the number of roots from the candidate's **cubic** graph. A1

# Mathematics C2 January 2009

# Solutions and Mark Scheme

# Final Version

**1.**

| | | | |
|---|---|---|---|
| 0 | 1·0 | | |
| 0·25 | 0·996108949 | | |
| 0·5 | 0·94117647 | | |
| 0·75 | 0·759643916 | (3 values correct) | B1 |
| 1 | 0·5 | (5 values correct) | B1 |

Correct formula with $h = 0{\cdot}25$ M1

$$I \approx \frac{0{\cdot}25}{2} \times \{1{\cdot}0 + 0{\cdot}5 + 2(0{\cdot}996108949 + 0{\cdot}94117647 + 0{\cdot}759643916)\}$$

$I \approx 0{\cdot}861732333$

$I \approx 0{\cdot}862$ (f.t. one slip) A1

**Special case** for candidates who put $h = 0{\cdot}2$

| | | | |
|---|---|---|---|
| 0 | 1·0 | | |
| 0·2 | 0·998402555 | | |
| 0·4 | 0·975039001 | | |
| 0·6 | 0·885269121 | | |
| 0·8 | 0·709421112 | | |
| 1 | 0·5 | (all values correct) | B1 |

Correct formula with $h = 0{\cdot}2$ M1

$$I \approx \frac{0{\cdot}2}{2} \times \{1{\cdot}0 + 0{\cdot}5 + 2(0{\cdot}998402555 + 0{\cdot}975039001 + 0{\cdot}885269121 + 0{\cdot}709421112)\}$$

$I \approx 0{\cdot}863626357$

$I \approx 0{\cdot}864$ (f.t. one slip) A1

**2.** (*a*) $6(1 - \sin^2\theta) + \sin\theta = 4$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$6\sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (3\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{2}{3}, -\frac{1}{2}$ A1

$\theta = 41{\cdot}81°, 138{\cdot}19°, 210°, 330°$ (41·81°, 138·19°) B1

(210°) B1

(330°) B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $3x = 123{\cdot}00°, 303{\cdot}00°, 483{\cdot}00°,$ (one value) B1

$x = 41{\cdot}00°, 101{\cdot}00°, 161{\cdot}00°,$ (one value) B1

(three values) B1

Note: Subtract 1 mark for each additional root in range, ignore roots outside range.

**3.** (*a*) $9^2 = 7^2 + x^2 - 2 \times 7 \times x \times {}^2/_7$ (correct substitution in cos rule) M1

$x^2 - 4x - 32 = 0$ A1

$x = 8$ (f.t. one slip in simplified quadratic) A1

(*b*) (i) Use of $\sin^2 B\hat{A}C = 1 - \cos^2 B\hat{A}C$ M1

$\sin B\hat{A}C = \frac{\sqrt{45}}{7}$ A1

(ii) $\frac{\sin ACB}{7} = \frac{\sin B\hat{A}C}{9}$ (correct use of sin rule) m1

$\sin ACB = \frac{\sqrt{45}}{9} = \frac{\sqrt{5}}{3}$ (c.a.o.) A1

**4.** (*a*) $a + 12d = 51$ B1

$a + 8d = k \times (a + d)$ ( $k = 5, {}^1/_5$) M1

$a + 8d = 5(a + d)$ A1

$3d = 4a$

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$d = 4, a = 3$ (both values) (c.a.o.) A1

(*b*) $S_{20} = \frac{20}{2} \times (5 + 62)$ (substitution of values in formula for sum of A.P.) M1

$S_{20} = 670$ A1

**5.** (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by *r* and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) $r = 0{\cdot}9$ B1

$S_{18} = \frac{10(1 - 0{\cdot}9^{18})}{1 - 0{\cdot}9}$ (f.t. candidate's numerical value for *r*) M1

$S_{18} \approx 84{\cdot}990 = 85$ (c.a.o.) A1

(*c*) (i) $ar = -4$ B1

$\frac{a}{1 - r} = 9$ B1

An attempt to solve these equations simultaneously by eliminating *a* M1

$9r^2 - 9r - 4 = 0$ (convincing) A1

(ii) $r = -{}^1/_3$ (c.a.o.) B1

$| r | < 1$ E1

**6.** (*a*) $3 \times \frac{x^{-1}}{-1} - 2 \times \frac{x^{3/2}}{3/2} + c$ (Deduct 1 mark if no *c* present) B1,B1

(*b*) (i) $5x - 4 - x^2 = 0$ M1

An attempt to solve quadratic equation in *x*, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 4$ (o.e.) m1

$(x - 1)(x - 4) = 0 \Rightarrow x = 1, x = 4$ (both values, c.a.o.) A1

(ii) Total area $= \int_1^4 (5x - 4 - x^2)\,dx - \int_4^5 (5x - 4 - x^2)\,dx$

(use of integration) M1

(subtraction of integrals with correct use of candidate's $x_A$, $x_B$ and 5 as limits) m1

$= \left[(5/2)x^2 - 4x - (1/3)x^3\right]_1^4 - \left[(5/2)x^2 - 4x - (1/3)x^3\right]_4^5$

(correct integration) B3

$= \{[40 - 16 - 64/3] - [5/2 - 4 - 1/3)]\}$
$\quad - \{[125/2 - 20 - 125/3] - [40 - 16 - 64/3]\}$

(substitution of candidate's limits in at least one integral) m1

$= 19/3$ (c.a.o.) A1

**7.** (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indicies) B1

$\log_a xy = p + q$ (relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) $\log_9 x = -\frac{1}{2} \Rightarrow x = 9^{-1/2}$

(rewriting log equation as power equation) M1

$x = 9^{-1/2} \Rightarrow x = \frac{1}{3}$ A1

(*c*) $2\log_a 3 = \log_a 3^2$ (power law) B1

$\log_a x + 2\log_a 3 = \log_a (3^2 \times x)$ (addition law) B1

$4x + 7 = 3^2 \times x$ (removing logs) M1

$x = 1{\cdot}4$ (c.a.o.) A1

| | | | |
|---|---|---|---|
| **8.** | (*a*) | $A(-2, 1)$ | B1 |
| | | A correct method for finding the radius | M1 |
| | | Radius = 5 | A1 |
| | (*b*) | An attempt to substitute $(6 - x)$ for $y$ in the equation of the circle | M1 |
| | | $x^2 - 3x + 2 = 0$ (or $2x^2 - 6x + 4 = 0$) | A1 |
| | | $x = 1, x = 2$ (correctly solving candidate's quadratic, both values) | A1 |
| | | Points of intersection are (1, 5), (2, 4) (c.a.o.) | A1 |
| | (*c*) | Distance between centres of $C_1$ and $C_2 = 13$ | B1 |
| | | Use of the fact that distance between centres = sum of the radii | M1 |
| | | $r = 8$ (c.a.o.) | A1 |
| **9.** | (*a*) | Substitution of values in area formula for triangle | M1 |
| | | Area = $\frac{1}{2} \times 4{\cdot}8^2 \times \sin 0{\cdot}7 = 7{\cdot}42$ cm$^2$. | A1 |
| | (*b*) | Let $R\hat{O}Q = \varphi$ radians | |
| | | $4{\cdot}8 \times \varphi = L$, $\frac{1}{2} \times 4{\cdot}8^2 \times \varphi = A$ (at least one correct equation) | B1 |
| | | An attempt to eliminate $\varphi$ | M1 |
| | | $k = 2{\cdot}4$ | A1 |

**A Level Mathematics C3**
**January 2009**
**Marking Scheme**

**1.** $h = \dfrac{\frac{2\pi}{9}}{4} = \dfrac{\pi}{18}$ — M1 (correct formula with $h = \pi/18$)

Integral = $\dfrac{\pi}{3 \times 18}$ [ 0 + (–0.26651509) + 4 (–0.01530883 – 0.14384104)
+ 2(–0.06220246)] — B1 (3 values)
B1 (other 2 values)

$\approx -0.0598$ — A1 (F.T. one slip)

$\int_0^{\frac{2\pi}{9}} \ln(\cos^2 x)\,dx \approx 2(-0.0598) = -0.1196$ — B1

**(5)**

**2.** (a) $\theta = 0$, $\cos 2\theta = 1$, for example — B1 (choice of $\theta$ and one correct evaluation)
$2\cos^2\theta - \sin^2\theta = 2$ — B1

(statement is false)

(b) $3(\sec^2\theta - 1) = 7 + \sec\theta$ — M1 (use of correct formula)
$3\sec^2\theta - \sec\theta - 10 = 0$ — M1 (attempt to solve quadratic, or correct formula or
$(3\sec\theta + 5)(\sec\theta - 2) = 0$ — $(a\sec\theta + b)(c\sec\theta + d)$ with $ac = 3$ $bd = -10$)

$\sec\theta = -\dfrac{5}{3}, 2$

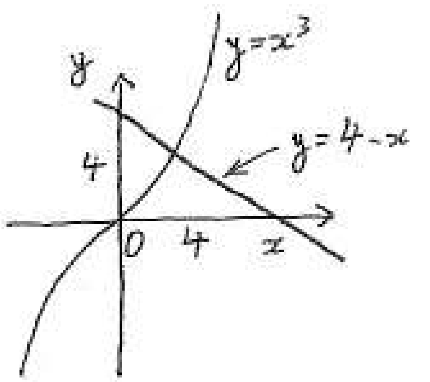
$\cos\theta = -\dfrac{3}{5}, \dfrac{1}{2}$ — A1 (values of $\cos\theta$)
$\theta = 126.9^\circ, 233.1^\circ, \quad 60^\circ, 300^\circ$ — B1 (126.9°) B1 (233.1°)
(allow to nearest degree) — B1 (60°, 300°)

**(8)**

| | | | |
|---|---|---|---|
| **3.** | (a) | $2x + 3x\dfrac{dy}{dx} + 3y + 4y\dfrac{dy}{dx} - 2 = 0$ | B1 $\left(3x\dfrac{dy}{dx} + 3y\right)$ $(o.e)$ |
| | | $2 + 3\dfrac{dy}{dx} + 6 + 8\dfrac{dy}{dx} - 2 = 0$ | B1 $\left(4y\dfrac{dy}{dx}\right)$ $(o.e)$ |
| | | | B1 (correct diff $^n$ of $x^2, -2x$ and 13) |
| | | $\dfrac{dy}{dx} = -\dfrac{6}{11}$ | B1 (F.T. one slip) |
| | (b) | $\dfrac{dy}{dx} = \dfrac{8e^{2t} + 3e^{t}}{2e^{t}}$ | M1 |
| | | | B1 $\left(\text{numerator } ke^{2t} + 3e^{t}, _{k=4,8}\right)$ |
| | | | B1 ( $k$ = 8)<br>B1 (denominator) |
| | | $\dfrac{8e^{2t} + 3e^{t}}{2e^{t}} = 6$ | M1 |
| | | $8e^{t} = 9$ | M1 |
| | | $t = ln\left(\dfrac{9}{8}\right) \approx 0.118$ | A1 (C.A.O) |

**(11)**

**4.** (a)




B1 $(y = x^3)$
B1 $(y = 4 - x)$

B1 one real root
∵ one intersection

(b) $x_0 = 1.4,\ x_1 = 1.37506.....,\ x_2 = 1.37945........$ — B1 $(x_1)$

$x_3 = 1.37868....,\ x_4 = 1.37881.... \approx 1.3788$ — B1 ($x_4$ 4 places)

Check 1.37875, 1.375885

| <u>x</u> | <u>f(x)</u> |
|---|---|
| 1.37875 | – 0.00031 |
| 1.37885 | ·0.0036 |

M1 (attempt to find signs or values)

A1 (correct)

Changes of sign indicates presence of root which is 1.3788 correct to 4 dec. places — A1 (conclusion)

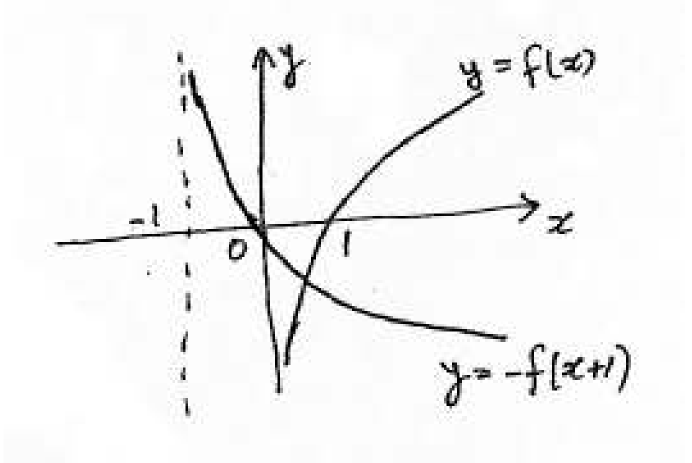
**(8)**

| | | | | |
|---|---|---|---|---|
| **5.** | (a) | (i) | $\frac{1}{\sin x}$ x $\cos x$ | M1 $\left(\frac{f(x)}{\sin x},\ f(x) = \pm \cos x\right)$ |
| | | | $= \cot x$ | A1 $(f(x) = \cos x)$ A1 $(\cot x)$ |
| | | | | $\left(accept\ \frac{1}{\tan x}\right)$ |
| | | (ii) | $\frac{4}{\sqrt{1-(4x)^2}}$ (o.e.) | M1 $\frac{k}{\sqrt{1-(4x)^2}}$ |
| | | | | A1 $(k = 4)$ |
| | | (iii) | $\frac{(x^2+5)(6x)-(3x^2+2)(2x)}{(x^2+5)^2}$ | M1 $\left(\frac{(x^2+5)f(x)-(3x^2+2)g(x)}{(x^2+5)^2}\right)$ |
| | | | | A1 $(f(x) = 6x,\ g(x) = 2x)$ |
| | | | $= \frac{26x}{(x^2+5)^2}$ | A1 |
| | (b) | $x = \tan y$ | | |
| | | $1 = \sec\ y^2 \frac{dy}{dx}$ | | M1 $\left(l = f(y)\ \frac{dy}{dx},\ f(y) \neq k\right)$ |
| | | | | A1 $(f(y) = \sec^2 y)$ |
| | | $\frac{dy}{dx} = \frac{1}{\sec^2 y}$ | | |
| | | $= \frac{1}{1+\tan^2 y}$ | | A1 |
| | | $= \frac{1}{1+x^2}$ | | A1 (C.A.O) |

**(12)**

| | | | |
|---|---|---|---|
| **6.** | (a) | $2\lvert x\rvert + 9 = 5\lvert x\rvert + 5$ | |
| | | $3\lvert x\rvert = 4$ | B1 $\begin{pmatrix} a\lvert x\rvert = b \\ a = 3,\ b = 4 \end{pmatrix}$ |
| | | $x = \pm \frac{4}{3}$ | B1 (both values) (F.T. $a, b$) |
| | (b) | $5x + 7 \le -4,\ x \le -\frac{3}{5}$ | B1 |
| and | | $5x + 7 \ge -4$ | M1 |
| | | $x \ge -\frac{11}{5}$ | |
| | | $-\frac{11}{5} \le x \le -\frac{3}{5}$ | A1 |
| | | | **(5)** |
| **7.** | (a) (i) | $\frac{7}{6}\ln\lvert 6x+5\rvert + c$ | M1 $(k\,ln\lvert 6x+5\rvert, k = 7, \frac{7}{6})$ |
| | | | A1 $\left(k = \frac{7}{6}\right)$ |
| | (ii) | $\frac{1}{5}\sin 5x + c$ | M1 $\left(k\ \sin 5x\ ,\ k = \pm\frac{1}{5}, 5, 1\right)$ |
| | | | A1 $\left(k = \frac{1}{5}\right)$ |
| | (b) | $\left[-\frac{9}{2(2x+1)}\right]_0^1$ | M1 $\left(\frac{k}{2x+1,}\ k = -9, \pm\frac{9}{2}\right)$ |
| | | | A1 $\left(k = -\frac{9}{2}\right)$ |
| | | $= -\frac{9}{2}\left[\frac{1}{3} - 1\right]$ | M1 $\left(k\left(\frac{1}{3} - 1\right)\right.$ allowable $k$ |
| | | $= 3$ | A1 $\left(\text{allow F.T for } k = \pm\frac{9}{2}\right)$ |
| | | | **(8)** |

**8.**

| | | |
|---|---|---|
| <br> | $y = f(x)$<br><br><br>$y = -f(x+1)$ | B1 (asymptote and shape)<br>B1 (1,0)<br>B1 (asymptote)<br>B1 (0,0)<br>B1 (behaviour, large $y$) |

**(5)**

**9.** (a) Let $y = 5x^2 + 4$

$y - 4 = 5x^2$ — M1 $(y-4=5x^2)$

$x = \pm\sqrt{\dfrac{y-4}{5}}$ — A2 (±) OR A1 (+) A1 $\left(\pm\dfrac{\sqrt{y-4}}{5}\right)$

$x = -\sqrt{\dfrac{y-4}{5}}$ — since domain $x \le 0$ — A1

$f^{-1}(x) = -\sqrt{\dfrac{x-4}{5}}$ — (F.T $x = f(y)$) — A1

(b) domain $x \ge 4$, Range $x \le 0$ (o.e) — B1

**(6)**

**10.** (a) Range of $f(x) \ge 2 - k$ (o.e) — B1

(b) $2 - k \ge 0$ — B1

$k \le 2$ — B1

(Greatest value of $k$ is 2)

(c) $3(4-k)^2 + 4 = 31$ — M1 (attempt to form equation, correct order, un......)

$(4-k)^2 = 9$ — A1

$k = 1, 7$ — A1

$\therefore k = 1$ — A1 (F.T max value of $k$ from (b))

(since $k \le 2$)

**(7)**

# Mathematics FP1 January 2009

## Solutions and Mark Scheme

## Final Version

1 (a) $\ln y = x \ln 2$ B1

$$\frac{1}{y}\frac{dy}{dx} = \ln 2$$ B1

$$\frac{dy}{dx} = y \ln 2 = 2^x \ln 2$$ B1

(b) $$f(x+h) - f(x) = \frac{x+h}{x+h+1} - \frac{x}{x+1}$$ M1

$$= \frac{(x+1)(x+h) - x(x+h+1)}{(x+h+1)(x+1)}$$ A1

$$= \frac{x^2 + x + hx + h - x^2 - hx - x}{(x+h+1)(x+1)}$$ A1

$$= \frac{h}{(x+h+1)(x+1)}$$ A1

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{h}{h(x+h+1)(x+1)}$$ M1

$$= \frac{1}{(x+1)^2}$$ A1

2 $$S_n = \sum_{r=1}^{n} (2r-1)^2$$ M1

$$= 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$ A1

$$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$$ A1A1A1

$$= \frac{n}{3}\left[4n^2 + 6n + 2 - 6n - 6 + 3\right]$$

$$= \frac{n(4n^2 - 1)}{3}$$

$$= \frac{n(2n+1)(2n-1)}{3}$$ cao A1

3

$$\alpha + \beta + \gamma = -4$$
$$\beta\gamma + \gamma\alpha + \alpha\beta = 3$$
$$\alpha\beta\gamma = -2 \qquad \text{B1}$$

Consider
` $$\beta\gamma + \gamma\alpha + \alpha\beta = 3 \qquad \text{B1}$$
$$\beta\gamma^2\alpha + \gamma\alpha^2\beta\gamma + \alpha\beta^2\gamma = \alpha\beta\gamma(\alpha + \beta + \gamma) \qquad \text{M1}$$
$$= 8 \qquad \text{A1}$$
$$\beta\gamma.\gamma\alpha.\alpha\beta = \alpha^2\beta^2\gamma^2 \qquad \text{M1}$$
$$= 4 \qquad \text{A1}$$

The required equation is
$$x^3 - 3x^2 + 8x - 4 = 0 \qquad \text{B1}$$
[FT on candidates earlier results]

4 (a) $2(x + \mathrm{i}y) - \mathrm{i}(x - \mathrm{i}y) = 1 + 4\mathrm{i}$ B1

$2x - y + \mathrm{i}(2y - x) = 1 + 4\mathrm{i}$ B1B1

Equating real and imaginary parts, M1

$2x - y = 1$

$2y - x = 4$ A1

The solution is $x = 2$, $y = 3$ ($z = 2 + 3\mathrm{i}$) A1A1

(b) $$\frac{1+3\mathrm{i}}{2-\mathrm{i}} = \frac{(1+3\mathrm{i})(2+\mathrm{i})}{(2-\mathrm{i})(2+\mathrm{i})} \qquad \text{M1}$$
$$= \frac{-1+7\mathrm{i}}{5} \qquad \text{A1A1}$$
$$|z| = \frac{\sqrt{1+49}}{5} = \sqrt{2} \qquad \text{B1}$$

Arg($z$) = 1.71 rad (98.1°) M1A1

5 (a) Fixed points satisfy
$$\begin{bmatrix} 0.6 & 0.8 & 2 \\ -0.8 & 0.6 & 3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \text{M1}$$
giving
$$0.4x - 0.8y = 2$$
$$0.8x + 0.4y = 3 \qquad \text{A1}$$

The solution is $(x, y) = \left(4, -\frac{1}{2}\right)$. cao A1A1

(b) The centre is $\left(4, -\frac{1}{2}\right)$ [FT from (a)] B1

The angle of rotation satisfies M1
$$\sin\theta = -\frac{4}{5}, \cos\theta = \frac{3}{5} \qquad \text{A1}$$
$$\theta = -53.1^\circ \text{ or } 306.9^\circ \qquad \text{A1}$$

6 The statement is true for $n = 1$ since putting $n = 1$, we obtain

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

which is correct. B1

Let the statement be true for $n = k$, ie

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & 2k & 2k^2 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{M1}$$

Consider

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^k \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{M1}$$

$$= \begin{bmatrix} 1 & 2k & 2k^2 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{A1}$$

$$= \begin{bmatrix} 1 & 2+2k & 2+4k+2k^2 \\ 0 & 1 & 2+2k \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{M1A1}$$

$$= \begin{bmatrix} 1 & 2(k+1) & 2(k+1)^2 \\ 0 & 1 & 2(k+1) \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{A1}$$

Thus true for $n = k \Rightarrow$ true for $n = k + 1$, hence proved by induction. A1

7 Let

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \text{M1}$$

$$\det(\mathbf{A}) = ad - bc \qquad \text{A1}$$

$$k\mathbf{A} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \qquad \text{A1}$$

Consider

$$\det(\text{k}\mathbf{A}) = k^2ad - k^2bc$$

$$= k^2(ad - bc) \qquad \text{A1}$$

$$= k^2 \det(\mathbf{A})$$

| | | | |
|---|---|---|---|
| 8 | (a) | $u + \mathrm{i}v = x + \mathrm{i}y - (x + \mathrm{i}y)^2$ or $(x + \mathrm{i}y)(1 - x - \mathrm{i}y)$ | M1 |
| | | $= x + \mathrm{i}y - (x^2 + 2\mathrm{i}xy - y^2)$ | A1 |
| | | Equating real and imaginary parts, | M1 |
| | | $v = y(1-2x)$<br>$u = x - x^2 + y^2$ | A1 |
| | (b) | Putting $y = x$, | M1 |
| | | $v = x(1-2x)$<br>$u = x$ | A1 |
| | | Eliminating $x$, the equation of the locus of $Q$ is | M1 |
| | | $v = u(1-2u)$ | A1 |
| 9 | (a)(i) | $\det(\mathbf{A}) = (\lambda+1)(2-\lambda^2) + 1(2\lambda - 1) + \lambda(\lambda - 4)$ | M1A1 |
| | | $= 1 - \lambda^3$ | A1 |
| | (ii) | When $\lambda = 1$, $\det(\mathbf{A}) = 0$ so $\mathbf{A}$ is singular. | B1 |
| | | Since 1 is known to have only one real cube root, we know that $\lambda = 1$ is the only value of $\lambda$ for which $\mathbf{A}$ is singular (or by factorising the cubic and showing the other roots are complex). | B1 |
| | (b)(i) | The equations are | |

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

The equations are consistent because the first and third rows are identical. B1

Put $z = \alpha$. M1

Then $y = \dfrac{4-\alpha}{3}$, $x = \dfrac{1-\alpha}{3}$ A1

(ii) $$\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

Cofactor matrix = $\begin{bmatrix} 1 & -3 & -5 \\ 0 & 2 & 2 \\ 1 & -1 & -1 \end{bmatrix}$ B1

Adjugate matrix = $\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}$ B1

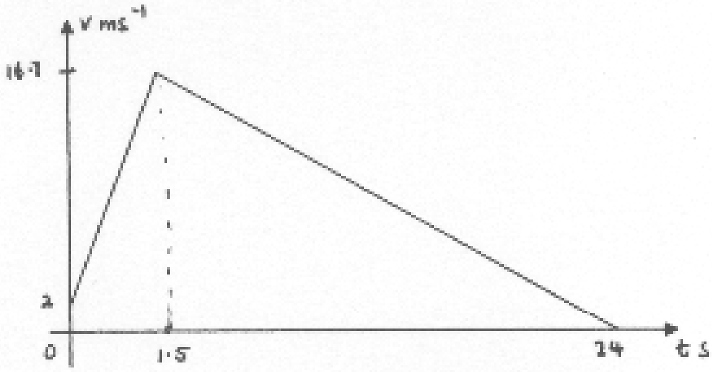
From (a), determinant = 2

$\mathbf{A}^{-1} = \dfrac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}$ B1

$\mathbf{X} = \dfrac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$ B1

| | | | | |
|---|---|---|---|---|
| 1. | (a) | Using $s = \frac{1}{2}(u+v)t$ with $s = 1200$ (2500), $v = 26$, $t = 60$. | oe | M1 |
| | | $1200 = \frac{1}{2}(u + 26)\,60$ | | A1 |
| | | $u = \underline{14 \text{ ms}^{-1}}$ | cao | A1 |
| | (b) | Using $v = u + at$ with $v = 26$, $u = 14$(c), $t = 60$. | oe | M1 |
| | | $26 = 14 + 60a$ | ft $u$ | A1 |
| | | $a = \underline{0.2 \text{ ms}^{-2}}$ | ft $u$ | A1 |
| | (c) | Using $v^2 = u^2 + 2as$ with $u = 26$ ($u = 14$c), $a = 0.2$(c), $s = 2500$ (3700). | oe | M1 |
| | | $v^2 = 26^2 + 2 \times 0.2 \times 2500$ | ft $a$ | A1 |
| | | $v = \underline{40.9 \text{ ms}^{-1}}$ | ft $a$ | A1 |
| 2. | (a) | Using $v = u + at$ with $u = 2$, $a = (\pm)9.8$, $t = 1.5$. | | M1 |
| | | $v = 2 + 9.8 \times 1.5$ | | A1 |
| | | $v = \underline{16.7 \text{ ms}^{-1}}$ | | A1 |

(b)

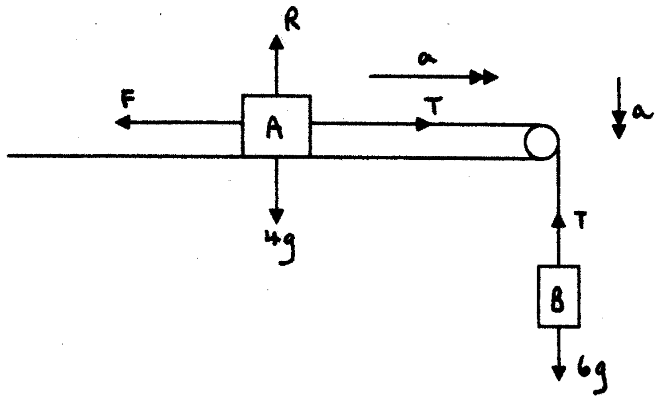



| | |
|---|---|
| (0,2) to (1.5,16.7c) | M1 |
| (0,2) to (1.5,16.7) to (24,0) | A1 |
| axes, labels and units | A1 |

| | | | | |
|---|---|---|---|---|
| (c) | Height = distance travelled | used | | M1 |
| | $= 0.5(2 + 16.7) \times 1.5 + 0.5 \times 16.7 \times 22.5$ | ft $v$ | | B1 |
| | $= \underline{201.9 \text{ m}}$ | ft ✓ | | A1 |

| | | | | |
|---|---|---|---|---|
| 3. | (a) | N2L | $15g - R = 15a$ dim correct, 15g andR opposing | M1 A1 |
| | | $a = -2$ | $R = 15 \times 9.8 - 15 \times (-2)$ | |
| | | | $R = \underline{177 \text{ N}}$ | A1 |
| | (b) | | $R = 15g = \underline{147 \text{ N}}$ | B1 |

4.



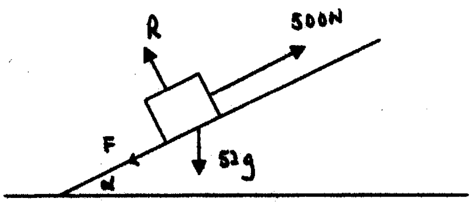

| | | |
|---|---|---|
| $R = 4g$ | si | B1 |
| $F = \mu R = 0.3 \times 4g = 1.2g$ | ft $R$ | B1 |

N2L applied to each mass

| | | |
|---|---|---|
| $6g - T = 6a$ | | M1 A1 |
| $T - F = 4a$ | | M1 A1 |

| | | | |
|---|---|---|---|
| Adding | $6g - 1.2g = 10a$ | both Ms | m1 |
| | $a = \dfrac{(6-1.2)\times 9.8}{10}$ | | |
| | $= \underline{4.704 \text{ ms}^{-2}}$ | ft slip in $R$ | A1 |
| | $T = 4\times 4\cdot 704 + 1\cdot 2\times 9\cdot 8$ | | |
| | $= \underline{30.576 \text{ N}}$ | ft slip in $R$ | A1 |

5.

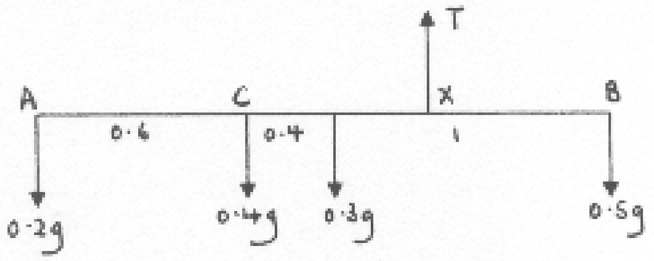



| | | |
|---|---|---|
| $R = 52g\cos\alpha$ | | M1 A1 |
| $= 470.4$ N | | |
| $F = \mu R$ | | M1 |
| $= 188.16$ N | | |

| | | |
|---|---|---|
| N2L | dim correct, all forces | M1 |
| $500 - F - mg\sin\alpha = ma$ | | A1 |
| $500 - 188.16 - 196 = 52a$ | | |
| $a = \underline{2.23\text{ms}^{-2}}$ | cao | A1 |

| | | | | |
|---|---|---|---|---|
| 6. | (a) | $e = \dfrac{2.8}{4}$ | | M1 |
| | | $e = \underline{0.7}$ | | A1 |
| | (b) | $I = 3(4 + 2.8)$ | | M1 |
| | | $I = \underline{20.4 \text{ Ns}}$ | | A1 |
| | (c) | Conservation of momentum | | M1 |
| | | $3 \times 2.8 + 5 \times 1.5 = 3v_A + 5v_B$ | | A1 |
| | | $3v_A + 5v_B = 15.9$ | | |
| | | Restitution | | M1 |
| | | $v_B - v_A = -0.6(1.5 - 2.8)$ = 0.78 | | A1 |
| | | $-3v_A + 3v_B = 2.34$ | | |
| | | Adding $8v_B = 18.24$ | dep on both Ms | m1 |
| | | $v_B = \underline{2.28 \text{ ms}^{-1}}$ | cao | A1 |
| | | $v_A = \underline{1.5 \text{ ms}^{-1}}$ | cao | A1 |

7.




| | | | |
|---|---|---|---|
| (a) | $T = (0.2 + 0.4 + 0.3 + 0.5)g$ | | M1 A1 |
| | $= 1.4g$ | | |
| | $= \underline{13.72 \text{ N}}$ | | A1 |
| (b) | Moments about $A$ | dim correct to obtain equation | M1 |
| | $Tx = 0.4g \times 0.6 + 0.3g \times 1 + 0.5g \times 2$ | ft $T$ | B1 A1 |
| | $1.4x = 0.24 + 0.3 + 1 = 1.54$ | | |
| | $x = \underline{1.1 \text{ m}}$ | cao | A1 |

| | | |
|---|---|---|
| 8. Resolve horizontally | | M1 |
| $T_X \cos 23° = T_Y \cos 30°$ | | A1 |
| $T_Y = \frac{2\cos 23°}{\sqrt{3}} T_X$ | | |
| Resolve vertically | | M1 |
| $T_X \sin 23° + T_Y \cos 60° = 12g$ | | A1 |
| $T_X\left(\sin 23° + \frac{\cos 23°}{\sqrt{3}}\right) = 12g$ | dep on both Ms | m1 |
| $T_X = \underline{127.52 \text{ N}}$ | cao | A1 |
| $T_Y = \underline{135.55 \text{ N}}$ | cao | A1 |

9. (a)

| | Area | from $AD$ | from$AB$ |
|---|---|---|---|
| $ABCD$ | 30 | 2.5 | 3 |
| $XYZ$ | 3 | 3 | 2 |
| Lamina | 27 | $x$ | $y$ |

| | | |
|---|---|---|
| Moments about $AD$ | | M1 |
| $30 \times 2.5 = 3 \times 3 + 27x$ | | A1 |
| $x = \frac{66}{27} = \frac{22}{9}$ | | |
| $x = 2\frac{4}{9}$ | cao | A1 |
| Moments about $AB$ | | M1 |
| $30 \times 3 = 3 \times 2 + 27y$ | | A1 |
| $y = \frac{84}{27} = \frac{28}{9}$ | | |
| $y = 3\frac{1}{9}$ | cao | A1 |
| (b) $\theta = \tan^{-1}\left(\frac{5 - \frac{22}{9}}{\frac{28}{9}}\right)$ | | M1 A1 |
| $= \tan^{-1}\left(\frac{23}{28}\right)$ | | |
| $= \underline{39.4°}$ | ft $x$, $y$ | A1 |
| (c) Required distance $= \frac{22}{9} = 2\frac{4}{9}$ | ft $x$ | B1 |

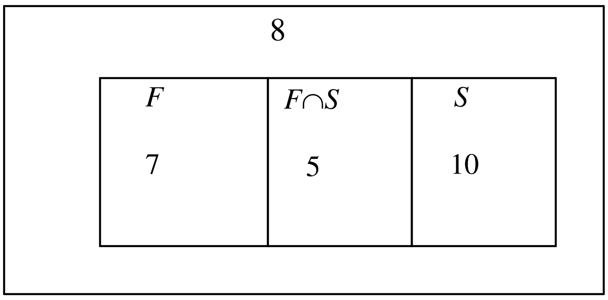
# Mathematics S1 January 2009

# Solutions and Mark Scheme

# Final Version

| | | | |
|---|---|---|---|
| 1 | (a) | Using $P(A \cup B) = P(A) + P(B)$ | M1 |
| | | $0.93 = 0.65 + P(B)$ so $P(B) = 0.28$ | A1 |
| | (b) | Using $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ | M1 |
| | | $0.93 = 0.65 + \mathrm{P(B)} - 0.65\mathrm{P(B)}$ | A1 |
| | | $0.35\mathrm{P(B)} = 0.28$ | M1 |
| | | $\mathrm{P(B)} = 0.8$ | A1 |
| 2 | EITHER | | |
| | (a) | $P(F \cup S) = 1 - P(F' \cap S')$ | M1 |
| | | $= 1 - 8/30 = 22/30$ | A1 |
| | | $P(F \cap S) = P(F) + P(S) - P(F \cup S)$ | M1 |
| | | $= (12 + 15 - 22)/30 = 5/30$ (1/6) | A1 |
| | (b) | $P(F \cap S') = P(F) - P(F \cap S)$ | M1 |
| | | $= (12 - 5)/30 = 7/30$ | A1 |

OR




B4

| | | | |
|---|---|---|---|
| | (a) | $P(F \cap S) = \dfrac{5}{30}$ | B1 |
| | (b) | $P(F \cap S') = \dfrac{7}{30}$ | B1 |
| | | [FT on minor slip] | |
| 3 | (a)(i) | Prob = $\mathrm{e}^{-2.75} \times \dfrac{2.75^4}{4!} = 0.152$ | M1A1 |
| | (ii) | $\mathrm{P}(\le 2) = \mathrm{e}^{-2.75}\left(1 + 2.75 + \dfrac{2.75^2}{2}\right)$ ( = 0.481) | M1A1 |
| | | Reqd prob = 0.519 | A1 |
| | (b)(i) | Reqd prob = 0.8153 | M1A1 |
| | (ii) | Reqd prob = 0.6472 – 0.4232 or 0.5768 – 0.3528 | B1B1 |
| | | = 0.224 | B1 |

| | | | |
|---|---|---|---|
| 4 | (a) | $E(Y) = 3 \times 4 - 7 = 5$ | M1A1 |
| | | $Var(X) = 4$ si | B1 |
| | | $Var(Y) = 9 \times 4 = 36$ | M1A1 |
| | (b) | $P(Y > 0) = P(3X > 7)$ | M1 |
| | | $= P(X \geq 3)$ | A1 |
| | | $= 0.7619$ cao | A1 |

5

(a) EITHER

Total number of possibilities = $\binom{16}{3}$ (=560) B1

Number of possibilities = $\binom{4}{1} \times \binom{4}{3}$ (= 16) B1

Prob = $\frac{16}{560} (= \frac{1}{35})$ cao B1

OR

Prob = $\frac{4}{16} \times \frac{3}{15} \times \frac{2}{14} \times 4$ M1A1

$= \frac{1}{35}$ cao A1

(b) EITHER

Number of possibilities = $(4 \times 4 \times 4) \times 4$ (= 256) M1A1A1

Prob = $\frac{256}{560} (= \frac{16}{35})$ cao A1

OR

Prob = $(\frac{4}{16} \times \frac{4}{15} \times \frac{4}{14} \times 6) \times 4$ M1A1A1

$= \frac{16}{35}$ cao A1

| | | | |
|---|---|---|---|
| 6 | (a)(i) | Let number of red flowers = $X$. | |
| | | $X$ is B(20,0.6) si | B1 |
| | | $P(X = 10) = \binom{20}{10} \times 0.6^{10} \times 0.4^{10}$ | M1 |
| | | $= 0.117$ | A1 |
| | (ii) | Let number of non-red flowers = $Y$ so $Y$ is B(20,0.4) si | B1 |
| | | $P(X \geq 12) = P(Y \leq 8)$ | M1 |
| | | $= 0.5956$ | A1 |
| | (b) | Let number of failures be $U$. | |
| | | $U$ is B(80,0.04) which is approx Poi(3.2). si | B1 |
| | | $P(U < 5) = 0.7806$ | M1A1 |

7 (a) $P(\text{Sum} = 5) = \dfrac{4}{36} = \dfrac{1}{9}$ M1A1

(b)(i) $\text{P(Score = 5)} = \dfrac{1}{2} \times P(\text{Score} = 5 | \text{head}) + \dfrac{1}{2} \times P(\text{Score} = 5 | \text{tail})$ M1

$= \dfrac{1}{2} \times \dfrac{1}{6} + \dfrac{1}{2} \times \dfrac{1}{9}$ A1

$= \dfrac{5}{36}$ A1

(ii) $P(\text{head} | 5) = \dfrac{1/12}{5/36}$ (FT denominator from (i)) B1B1

$= \dfrac{3}{5}$ cao (but FT from (a)) B1

8 (a) $E(X) = \Sigma x P(X = x)$ M1

$= \dfrac{1}{20}(8 \times 2 + 6 \times 4 + 4 \times 6 + 2 \times 8))$ A1

$= 4$ A1

$E(X^2) = \Sigma x^2 P(X = x)$ M1

$= \dfrac{1}{20}(8 \times 2^2 + 6 \times 4^2 + 4 \times 6^2 + 2 \times 8^2)$ (= 20) A1

Variance = 20 – 16 = 4 cao A1

(b) The possibilities are 2 and 6 or 4 and 4. M1

$\text{Prob} = \dfrac{8}{20} \times \dfrac{4}{20} + \dfrac{4}{20} \times \dfrac{8}{20} + \dfrac{6}{20} \times \dfrac{6}{20}$ (Must have 3 terms) M1

$= \dfrac{1}{4}$ A1

| | | | |
|---|---|---|---|
| 9 | (a) | Since $F(2) = 1$, it follows that | M1 |
| | | $k \times 2^3 = 1 \Rightarrow k = \frac{1}{8}$ | A1 |
| | (b) | Prob = F(1.5) – F(0.5) | M1 |
| | | $= \frac{1}{8}\left(1.5^3 - 0.5^3\right) = \frac{13}{32}$ (0.406) | A1 |
| | (c) | The median $m$ satisfies | |
| | | $\frac{1}{8}m^3 = \frac{1}{2}$ | M1 |
| | | $m = \sqrt[3]{4}(= 1.59)$ | A1 |
| | (d) | The probability density $f(x)$ is given by | |
| | | $f(x) = F'(x) = \frac{3}{8}x^2$ | M1A1 |
| | | $E(X) = \int_0^2 \frac{3}{8}x^2 \times x\mathrm{d}x$ | M1A1 |
| | | $= \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2$ | A1 |
| | | $= \frac{3}{2}$ | A1 |

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