C3

Solutions and Mark Scheme

Final Version

1.

0	0·69314718		
0·25	0·825939419		
0·5	0·974076984		
0·75	1·136871006	(5 values correct)	B2
1	1·313261688	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{3} \times \{0{\cdot}69314718 + 1{\cdot}313261688 + 4(0{\cdot}825939419 + 1{\cdot}136871006) + 2(0{\cdot}974076984)\}$

$I \approx 11{\cdot}80580453 \div 12$

$I \approx 0{\cdot}983817044$

$I \approx 0{\cdot}984$ (f.t. one slip) A1

2. (*a*) e.g. $\theta = \frac{\pi}{2}$

$\sin 4\theta = 0$ (choice of θ and one correct evaluation) B1

$4\sin^3\theta - 3\sin\theta = 1$ (both evaluations correct but different) B1

(*b*) $3(1 + \tan^2\theta) = 7 - 11\tan\theta$. (correct use of $\sec^2\theta = 1 + \tan^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\tan\theta$, either by using the quadratic formula or by getting the expression into the form $(a\tan\theta + b)(c\tan\theta + d)$, with $a \times c$ = coefficient of $\tan^2\theta$ and $b \times d$ = constant m1

$3\tan^2\theta + 11\tan\theta - 4 = 0 \Rightarrow (3\tan\theta - 1)(\tan\theta + 4) = 0$

$\Rightarrow \tan\theta = \frac{1}{3}, \tan\theta = -4$ (c.a.o.) A1

$\theta = 18{\cdot}4°, 198{\cdot}4°$ B1

$\theta = 104{\cdot}0°, 284{\cdot}0°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta = +, -$, f.t. for 3 marks, $\tan\theta = -, -$, f.t. for 2 marks
$\tan\theta = +, +$, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$ B1

$\frac{d}{dx}(2x^3y) = 2x^3\frac{dy}{dx} + 6x^2y$ B1

$\frac{d}{dx}(3x^2 + 4x - 3) = 6x + 4$ B1

$x = 2, y = 1 \Rightarrow \frac{dy}{dx} = -\frac{8}{19}$ (c.a.o.) B1

(*b*) (i) $\frac{dx}{dt} = 6t$, $\frac{dy}{dt} = 12t^2 + 6t^5$ (all three terms correct) B2

(one term correct) B1

Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

$\frac{dy}{dx} = 2t + t^4$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 2 + 4t^3$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$ M1

$\frac{d^2y}{dx^2} = \frac{1 + 2t^3}{3t}$ (c.a.o.) A1

4. $f(x) = 2 - 10x + \sin x$

An attempt to check values or signs of $f(x)$ at $x = 0$, $x = \pi/8$ M1

$f(0) = 2 > 0$, $f(\pi/8) = -1{\cdot}54 < 0$

Change of sign $\Rightarrow f(x) = 0$ has root in $(0, \pi/8)$ A1

$x_0 = 0{\cdot}2$

$x_1 = 0{\cdot}219866933$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 0{\cdot}221809976$

$x_3 = 0{\cdot}221999561$

$x_4 = 0{\cdot}222018055 = 0{\cdot}22202$ (x_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 0{\cdot}222015$, $x = 0{\cdot}222025$ M1

$f(0{\cdot}222015) = 4{\cdot}56 \times 10^{-5} > 0$, $f(0{\cdot}222025) = -4{\cdot}46 \times 10^{-5} < 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}22202$ correct to five decimal places A1

Note: 'change of sign' must appear at least once

5. (*a*) $\frac{dy}{dx} = \frac{3}{1 + (3x)^2}$ **or** $\frac{1}{1 + (3x)^2}$ **or** $\frac{3}{1 + 3x^2}$ M1

$\frac{dy}{dx} = \frac{3}{1 + 9x^2}$ A1

(*b*) $\frac{dy}{dx} = \frac{ax + b}{2x^2 - 3x + 4}$ (including $a = 0$, $b = 1$) M1

$\frac{dy}{dx} = \frac{4x - 3}{2x^2 - 3x + 4}$ A1

(*c*) $\frac{dy}{dx} = e^{2x} \times m\cos x + ke^{2x} \times \sin x$ ($m = \pm 1$, $k = 1, 2$) M1

$\frac{dy}{dx} = e^{2x} \times m\cos x + ke^{2x} \times \sin x$ (either $m = 1$ or $k = 2$) A1

$\frac{dy}{dx} = e^{2x} \times \cos x + 2e^{2x} \times \sin x$ (c.a.o.) A1

(*d*) $\frac{dy}{dx} = \frac{(1 + \cos x) \times m\sin x - (1 - \cos x) \times k\sin x}{(1 + \cos x)^2}$ ($m = \pm 1$, $k = \pm 1$) M1

$\frac{dy}{dx} = \frac{(1 + \cos x) \times -(-\sin x) - (1 - \cos x) \times (-\sin x)}{(1 + \cos x)^2}$ A1

$\frac{dy}{dx} = \frac{2\sin x}{(1 + \cos x)^2}$ A1

6. (*a*) (i) $\int \frac{1}{4x-7}\,dx = k \times \ln|4x-7| + c \quad (k = 1, 4, {}^1/_4)$ M1

$\int \frac{1}{4x-7}\,dx = {}^1/_4 \times \ln|4x-7| + c$ A1

(ii) $\int e^{3x-1}\,dx = k \times e^{3x-1} + c \quad (k = 1, 3, {}^1/_3)$ M1

$\int e^{3x-1}\,dx = {}^1/_3 \times e^{3x-1} + c$ A1

(iii) $\int \frac{5}{(2x+3)^4}\,dx = -\frac{5}{3k} \times (2x+3)^{-3} + c \quad (k = 1, 2, {}^1/_2)$ M1

$\int \frac{5}{(2x+3)^4}\,dx = -\frac{5}{6} \times (2x+3)^{-3} + c$ A1

(*b*) $\int \sin\left(2x + \frac{\pi}{4}\right) dx = \left[k \times \cos\left(2x + \frac{\pi}{4}\right)\right] \quad (k = -1, -2, \pm {}^1/_2)$ M1

$\int \sin\left(2x + \frac{\pi}{4}\right) dx = \left[-{}^1/_2 \times \cos\left(2x + \frac{\pi}{4}\right)\right]$ A1

$\int_0^{\pi/4} \sin\left(2x + \frac{\pi}{4}\right) dx = k \times \left[\cos\left(\frac{3\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)\right]$

(f.t. candidate's value for k) A1

$\int_0^{\pi/4} \sin\left(2x + \frac{\pi}{4}\right) dx = \frac{\sqrt{2}}{2}$ (c.a.o.) A1

7. (*a*) $2|x+1| - 3 = 7 \Rightarrow |x+1| = 5$ B1

$x = 4, -6$ B1

(*b*) Trying to solve either $5x - 8 \geq 3$ or $5x - 8 \leq -3$ M1

$5x - 8 \geq 3 \Rightarrow x \geq 2{\cdot}2$

$5x - 8 \leq -3 \Rightarrow x \leq 1$ (both inequalities) A1

Required range: $x \leq 1$ or $x \geq 2{\cdot}2$ (f.t. one slip) A1

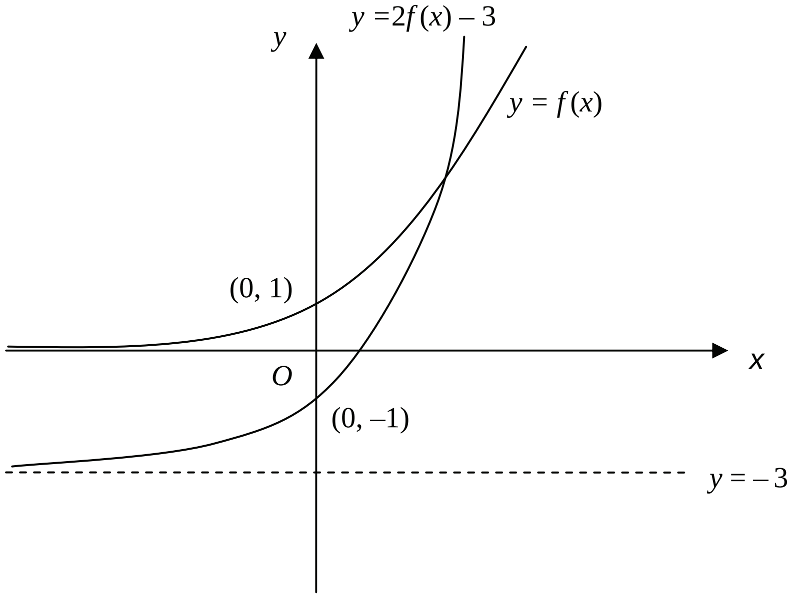
Alternative mark scheme

$(5x-8)^2 \geq 9$ (forming and trying to solve quadratic) M1

Critical points $x = 1$ and $x = 2{\cdot}2$ A1

Required range: $x \leq 1$ or $x \geq 2{\cdot}2$ (f.t. one slip in critical points) A1

8.



The x-axis is an asymptote for $f(x)$ at $-\infty$, correct behaviour at $+\infty$	M1
$y = f(x)$ cuts y-axis at (0, 1)	A1
$y = 2f(x) - 3$ cuts y-axis at (0, –1) (f.t. candidate's y-intercept for $f(x)$)	B1
$y = -3$ is an asymptote for $2f(x) - 3$ at $-\infty$, with graph above $y = -3$	B1
The diagram shows that the graph of $y = 2f(x) - 3$ is steeper than the graph of $y = f(x)$ in the first quadrant	B1

9. (*a*) $y = \frac{1}{2}\sqrt{x} - 3$ and an attempt to isolate x M1

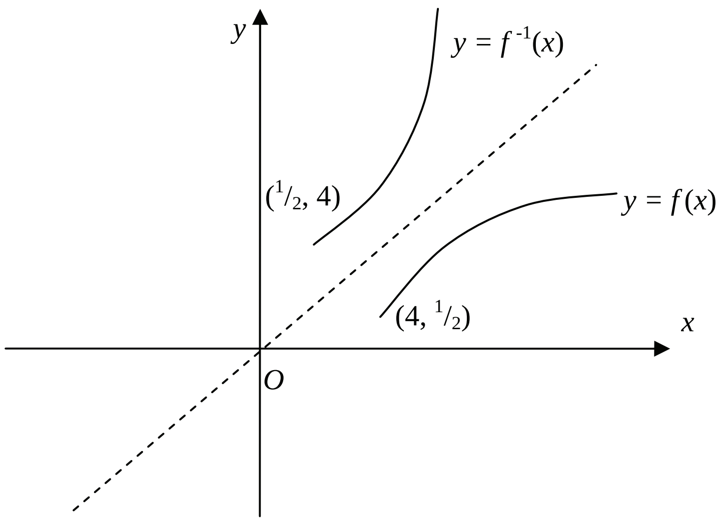
$2y = \sqrt{x} - 3 \Rightarrow x = 4y^2 + 3$ A1

$f^{-1}(x) = 4x^2 + 3$ (f.t. one slip in candidate's expression for x) A1

$R(f^{-1}) = [4, \infty)$ B1

$D(f^{-1}) = [{}^1/_2, \infty)$ B1

(*b*)



$y = f^{-1}(x)$ a parabola B1

starting at $({}^1/_2, 4)$ (f.t. candidate's $D(f^{-1})$) B1

$y = f(x)$ as in diagram (c.a.o.) B1

10. (*a*) $R(f) = (-1, \infty)$ B1

$R(g) = (3, \infty)$ B1

(*b*) $f(1) = 0$ is not in the domain of g E1

(*c*) (i) $fg(x) = (2x - 1)^2 - 1$ M1

$fg(x) = 4x(x - 1)$ or $4x^2 - 4x$ A1

(ii) $D(fg) = (2, \infty)$ B1

$R(fg) = (8, \infty)$ B1