GCE AS/A level

975/01

MATHEMATICS C3

Pure Mathematics

P.M. WEDNESDAY, 20 January 2010

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^1 \ln(1 + e^x)\,dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\sin 4\theta \equiv 4\sin^3\theta - 3\sin\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$3\sec^2\theta = 7 - 11\tan\theta.$$

Give your answers correct to one decimal place. [6]

3. *(a)* The curve C is defined by

$$y^3 + 2x^3y = 3x^2 + 4x - 3.$$

Find the value of $\frac{dy}{dx}$ at the point (2, 1). [4]

(b) Given that $x = 3t^2$, $y = 4t^3 + t^6$, find, in terms of t,

(i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

Simplify your answers. [7]

4. Show that the equation

$$2 - 10x + \sin x = 0$$

has a root α between 0 and $\frac{\pi}{8}$.

The recurrence relation

$$x_{n+1} = \frac{1}{10}\left(2 + \sin x_n\right) ,$$

with $x_0 = 0 \cdot 2$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

5. Differentiate **each** of the following with respect to x, simplifying your answer wherever possible.

(a) $\tan^{-1} 3x$ *(b)* $\ln(2x^2 - 3x + 4)$ [2], [2]

(c) $e^{2x} \sin x$ *(d)* $\dfrac{1 - \cos x}{1 + \cos x}$ [3], [3]

6. *(a)* Find

(i) $\displaystyle\int \frac{1}{4x-7}\,dx$, (ii) $\displaystyle\int e^{3x-1}\,dx$, (iii) $\displaystyle\int \frac{5}{(2x+3)^4}\,dx$. [6]

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin\left(2x + \frac{\pi}{4}\right) dx$, expressing your answer in surd form. [4]

7. Solve the following.

(a) $2|x + 1| - 3 = 7$ [2]

(b) $|5x - 8| \geqslant 3$ [3]

8. Given that $f(x) = e^x$, sketch, on the same diagram, the graphs of $y = f(x)$ and $y = 2f(x) - 3$. Label the coordinates of the point of intersection of each of the graphs with the y-axis. Indicate the behaviour of each of the graphs for large positive and negative values of x. [5]

9. The function f has domain $[4, \infty)$ and is defined by

$$f(x) = \frac{1}{2}\sqrt{x-3}\ .$$

(a) Find an expression for $f^{-1}(x)$. Write down the range and domain of f^{-1}. [5]

(b) Sketch the graph of $y = f^{-1}(x)$. On the same diagram, sketch the graph of $y = f(x)$. [3]

10. The functions f and g have domains $(0, \infty)$ and $(2, \infty)$ respectively and are defined by

$$f(x) = x^2 - 1,$$
$$g(x) = 2x - 1.$$

(a) Write down the ranges of f and g. [2]

(b) Give the reason why $gf(1)$ cannot be formed. [1]

(c) (i) Find an expression for $fg(x)$. Simplify your answer.

(ii) Write down the domain and range of fg. [4]