



GCE MARKING SCHEME

MATHEMATICS
AS/Advanced

JANUARY 2011

INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2011 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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C1

1. (a) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1
 Gradient of $AB = \frac{1}{3}$ (or equivalent) A1
- (b) A correct method for finding the equation of AB using the candidate's value for the gradient of AB . M1
 Equation of AB : $y - 2 = \frac{1}{3}[x - (-1)]$ (or equivalent) A1
 (f.t. the candidate's value for the gradient of AB)
 Equation of AB : $x - 3y + 7 = 0$
 (f.t. one error if both M1's are awarded) A1
- (c) A correct method for finding C M1
 $C(17, 8)$ A1
- (d) (i) An attempt to use the fact that gradient of $L =$ gradient of AB M1
 Equation of L : $y = \frac{1}{3}x - \frac{1}{6}$ (o.e.) A1
 (f.t. the candidate's value for the gradient of AB)
 (ii) Putting $y = 0$ in candidate's equation for L M1
 $D(0.5, 0)$ (f.t. candidate's equation for L) A1
 (iii) A correct method for finding the length of AD M1
 $AD = 2.5$ (c.a.o.) A1
2. $\frac{\sqrt{2}}{10 - 7\sqrt{2}} = \frac{\sqrt{2} \times (10 + 7\sqrt{2})}{(10 - 7\sqrt{2})(10 + 7\sqrt{2})}$ M1
 Numerator: $10\sqrt{2} + 14$ A1
 Denominator: $100 - 98$ A1
 $\frac{\sqrt{2}}{10 - 7\sqrt{2}} = 5\sqrt{2} + 7$ (c.a.o.) A1

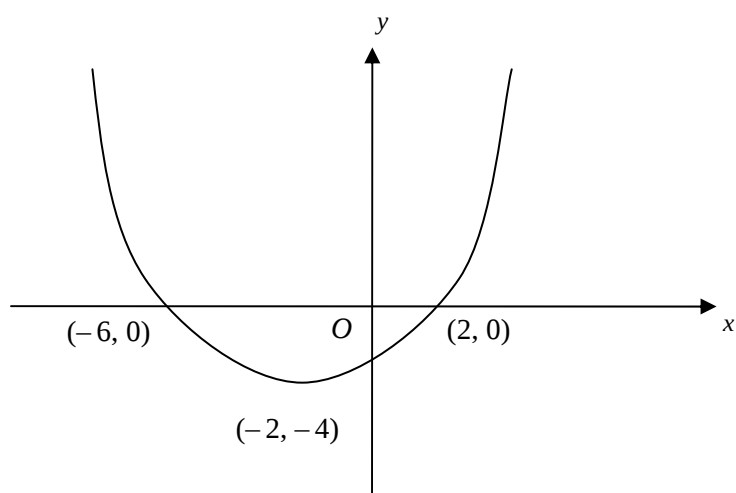
Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $10 - 7\sqrt{2}$

3. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1
 $b^2 - 4ac = (3k - 1)^2 - 4 \times 2 \times (3k^2 - 1)$ A1
 Putting $b^2 - 4ac > 0$ m1
 $5k^2 + 2k - 3 < 0$ (convincing) A1
 Finding critical values $k = -1, k = \frac{3}{5}$ B1
 $-1 < k < \frac{3}{5}$ **or** $\frac{3}{5} > k > -1$ **or** $(-1, \frac{3}{5})$ **or** $-1 < k$ and $k < \frac{3}{5}$ **or** a correctly worded statement to the effect that k lies strictly between -1 and $\frac{3}{5}$
 (f.t. only critical values of ± 1 and $\pm \frac{3}{5}$) B2
- Note:
 $-1 \leq k \leq \frac{3}{5}$
 $-1 < k, k < \frac{3}{5}$
 $-1 < k \leq \frac{3}{5}$
 $-1 \leq k < \frac{3}{5}$
 all earn B1
4. (a) $y + \delta y = 6(x + \delta x)^2 + 4(x + \delta x) - 9$ B1
 Subtracting y from above to find δy M1
 $\delta y = 12x\delta x + 6(\delta x)^2 + 4\delta x$ A1
 Dividing by δx and letting $\delta x \rightarrow 0$ M1
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 12x + 4$ (c.a.o.) A1
- (b) Required derivative $= 3 \times (-4) \times x^{-5} - 7 \times (\frac{1}{3}) \times x^{-2/3}$ B1, B1
5. $(1 + \sqrt{3})^5 = (1)^5 + 5(1)^4(\sqrt{3}) + 10(1)^3(\sqrt{3})^2 + 10(1)^2(\sqrt{3})^3 + 5(1)(\sqrt{3})^4 + (\sqrt{3})^5$
 (five or six terms correct) B2
 (four terms correct) B1
 $(1 + \sqrt{3})^5 = 1 + 5\sqrt{3} + 30 + 30\sqrt{3} + 45 + 9\sqrt{3}$
 (six terms correct) B2
 (four or five terms correct) B1
 $(1 + \sqrt{3})^5 = 76 + 44\sqrt{3}$ (f.t. one error) B1
6. Either $p = -0.7$ or a sight of $(x - 0.7)^2$ B1
 A convincing argument to show that the value 9 is correct B1
 $x^2 - 1.4x - 8.51 = 0 \Rightarrow (x - 0.7)^2 = 9$ M1
 $x = 3.7$ A1
 $x = -2.3$ A1

7. (a) An attempt to calculate $(-2)^3 - 3$ M1
 Remainder = -11 A1
- (b) Attempting to find $f(r) = 0$ for some value of r M1
 $f(-1) = 0 \Rightarrow x + 1$ is a factor A1
 $f(x) = (x + 1)(6x^2 + ax + b)$ with one of a, b correct M1
 $f(x) = (x + 1)(6x^2 - 5x - 6)$ A1
 $f(x) = (x + 1)(3x + 2)(2x - 3)$ (f.t. only $6x^2 + 5x - 6$ in above line) A1
 Roots are $x = -1, -\frac{2}{3}, \frac{3}{2}$ (f.t. for factors $3x \pm 2, 2x \pm 3$) A1
Special case
 Candidates who, after having found $x + 1$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1
8. (a) y -coordinate at $P = 2$ B1
 $\frac{dy}{dx} = 2x - 6$ (an attempt to differentiate, at least one non-zero term correct) M1
 An attempt to substitute $x = 5$ in candidate's expression for $\frac{dy}{dx}$ m1
 Use of candidate's numerical value for $\frac{dy}{dx}$ as gradient of tangent at P m1
 Equation of tangent at P : $y - 2 = 4(x - 5)$ (or equivalent) A1
 (f.t. only candidate's value for y -coordinate at P)
- (b) (i) $x^2 - 6x + 7 = \frac{1}{2}x - 2$ (o.e.) M1
 An attempt to collect terms, form and solve quadratic equation m1
 $2x^2 - 13x + 18 = 0 \Rightarrow (x - 2)(2x - 9) = 0 \Rightarrow x = 2, x = 4\frac{1}{2}$ (both values, c.a.o.) A1
 When $x = 2, y = -1$, when $x = 4\frac{1}{2}, y = \frac{1}{4}$ (both values f.t. one numerical slip) A1
 (ii) Values of $\frac{dy}{dx}$ at points of intersection of C and L are 3 and -2
 (at least one correct, f.t. candidate's derived x -coordinates at points of intersection of C and L) B1
 Use of the fact that
 gradient of normal = $-\frac{1}{\frac{dy}{dx}}$
 at at least one of the candidate's points of intersection of C and L M1
 Normal to C at point with x -coordinate 2 has gradient $\frac{1}{2}$ (c.a.o.) A1
 Since gradient of $L = \frac{1}{2}$, L and this normal must coincide A1

9. (a)



Concave up curve and y-coordinate of minimum = -4

B1

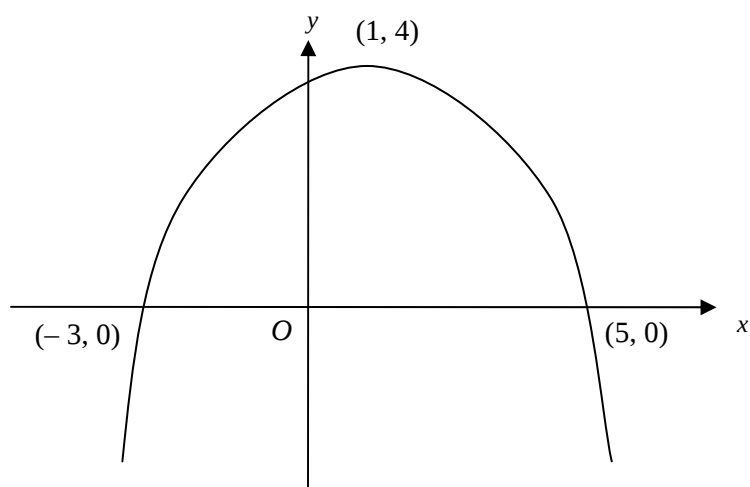
x-coordinate of minimum = -2

B1

Both points of intersection with x-axis

B1

(b)



Concave down curve and x-coordinate of maximum = 1

B1

y-coordinate of maximum = 4

B1

Both points of intersection with x-axis

B1

10. (a) $\frac{dy}{dx} = 3x^2 + 2kx - 9$ B1
 Putting derived $\frac{dy}{dx} = 0$ when $x = -1$ M1
 $3 - 2k - 9 = 0 \Rightarrow k = -3$ (convincing) A1
- (b) An attempt to solve $3x^2 - 6x - 9 = 0$ M1
 x -coordinate of R is 3 A1
- (c) A correct method for finding nature of stationary points yielding
either Q is a maximum point **or** R is a minimum point M1
 Correct conclusion for other point
 (f.t. candidate's value for x -coordinate of R) A1

C2

1.	1	2.236067977		
	1.25	2.439902662		
	1.5	2.715695123		
	1.75	3.059309563	(5 values correct)	B2
	2	3.464101615	(3 or 4 values correct)	B1
	Correct formula with $h = 0.25$			M1
	$I \approx \frac{0.25}{2} \times \{2.236067977 + 3.464101615 + 2(2.439902662 + 2.715695123 + 3.059309563)\}$			
	$I \approx 22.12998429 \times 0.25 \div 2$			
	$I \approx 2.766248036$			
	$I \approx 2.766$			(f.t. one slip) A1

Special case for candidates who put $h = 0.2$

	1	2.236067977		
	1.2	2.393324048		
	1.4	2.596921254		
	1.6	2.845347079		
	1.8	3.135602016		
	2	3.464101615	(all values correct)	B1
	Correct formula with $h = 0.2$			M1
	$I \approx \frac{0.2}{2} \times \{2.236067977 + 3.464101615 + 2(2.393324048 + 2.596921254 + 2.845347079 + 3.135602016)\}$			
	$I \approx 27.64255839 \times 0.2 \div 2$			
	$I \approx 2.764255839$			
	$I \approx 2.764$			(f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (a) $7 \sin^2 \theta + 1 = 3(1 - \sin^2 \theta) - \sin^2 \theta$
 (correct use of $\cos^2 \theta = 1 - \sin^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation in $\sin \theta$, either by using the quadratic formula or by getting the expression into the form $(a \sin \theta + b)(c \sin \theta + d)$, with $a \times c = \text{coefficient of } \sin^2 \theta$ and $b \times d = \text{candidate's constant}$ m1
 $10 \sin^2 \theta + \sin \theta - 2 = 0 \Rightarrow (2 \sin \theta + 1)(5 \sin \theta - 2) = 0$
 $\Rightarrow \sin \theta = -\frac{1}{2}, \sin \theta = \frac{2}{5}$ (c.a.o.) A1
 $\theta = 210^\circ, 330^\circ$ B1 B1
 $\theta = 23.58^\circ, 156.42^\circ$ B1
 Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
 $\sin \theta = +, -, \text{f.t. for 3 marks, } \sin \theta = -, -, \text{f.t. for 2 marks}$
 $\sin \theta = +, +, \text{f.t. for 1 mark}$
- (b) $2x + 25^\circ = 117^\circ, 243^\circ$, (one value) B1
 $x = 46^\circ, 109^\circ$ B1, B1
 Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.
3. (a) $(x + 6)^2 = x^2 + (x + 1)^2 - 2 \times x \times (x + 1) \times \cos 120^\circ$
 (correct use of cos rule) M1
 $2x^2 - 9x - 35 = 0$ (convincing) A1
 An attempt to solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c = 2$ and $b \times d = -35$ M1
 $(2x + 5)(x - 7) = 0 \Rightarrow x = 7$ A1
- (b) Area = $\frac{1}{2} \times 7 \times (7 + 1) \times \sin 120^\circ$
 (substituting the correct values in the correct places in the area formula, f.t. candidate's derived value for x) M1
 Area = 24.25 cm^2 (f.t. candidate's derived value for x) A1

4. (a) $S_n = a + [a + d] + \dots + [a + (n - 1)d]$ (at least 3 terms, one at each end) B1
 $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots + a$
 Either:
 $2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \dots + [a + a + (n - 1)d]$
 Or:
 $2S_n = [a + a + (n - 1)d] + \dots$ (n times) M1
 $2S_n = n[2a + (n - 1)d]$
 $S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1
- (b) $a + 7d = 28$ B1
 $\frac{20}{2} \times [2a + 19d] = 710$ B1
 An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1
 $d = 3$ (c.a.o.) A1
 $a = 7$ (f.t. candidate's value for d) A1
- (c) $S_{15} = \frac{15}{2} \times (-3 + 67)$ (substitution of values in formula for sum of A.P.) M1
 $S_{15} = 480$ A1
5. (a) (i) $ar = 6$ and $ar^4 = 384$ B1
 $r^3 = \frac{384}{6}$ (o.e.) M1
 $r = 4$ (c.a.o.) A1
 (ii) $a \times 4 = 6 \Rightarrow a = 1.5$ B1
 $S_8 = \frac{1.5(4^8 - 1)}{4 - 1}$ (correct use of formula for S_8 , f.t. candidate's derived values for r and a) M1
 $S_8 = 32767.5$ (f.t. candidate's derived values for r and a) A1
- (b) (i) $5 \times 1 \cdot 1^{n-1} = 170$ M1
 $1 \cdot 1^{n-1} = 34$ A1
 $(n - 1)\log 1 \cdot 1 = \log 34$ (f.t. only $5 \cdot 5^{n-1} = 170$ or $1 \cdot 1^n = 34$) M1
 $n = 38$ (c.a.o.) A1
 (ii) $|r|$ must be < 1 for sum to infinity to exist E1

6. (a) $3 \times \frac{x^{1/2}}{1/2} - 4 \times \frac{x^{5/3}}{5/3} + c$ B1, B1

(−1 if no constant term present)

(b) (i) $25 - x^2 = -2x + 17$ M1

An attempt to rewrite and solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b =$ candidate's constant m1

$(x - 4)(x + 2) = 0 \Rightarrow x = 4, -2$ (both values, c.a.o.) A1

$y = 9, y = 21$ (both values, f.t. candidate's x -values) A1

(ii) **Either:**

Total area $= \int_{-2}^4 (25 - x^2) dx - \int_{-2}^4 (-2x + 17) dx$
(use of integration) M1

(subtraction of integrals with correct use of candidate's x_A, x_B as limits) m1

$\int x^2 dx = \frac{x^3}{3}, \quad \int 2x dx = x^2$ B1 B1

Either: $\int 25 dx = 25x$ **and** $\int 17 dx = 17x$ or: $\int 8 dx = 8x$ B1

Total area $= [25x - (1/3)x^3]_{-2}^4 - [-x^2 + 17x]_{-2}^4$ (o.e.)

$= \{(100 - 64/3) - (-50 - (-8/3))\} - \{(-16 + 68) - (-4 - 34)\}$
(substitution of candidate's limits in at least one integral) m1
 $= 36$ (c.a.o.) A1

Or:

Area of trapezium $= 90$
(f.t. candidate's x -coordinates for A, B) B1

Area under curve $= \int_{-2}^4 (25 - x^2) dx$
(use of integration) M1

$= [25x - (1/3)x^3]_{-2}^4$
(correct integration) B2

$= \{(100 - 64/3) - (-50 - (-8/3))\}$
(substitution of candidate's limits) m1
 $= 126$

Use of candidate's, x_A, x_B as limits and trying to find total area by subtracting area of trapezium from area under curve m1
Total area $= 126 - 90 = 36$ (c.a.o.) A1

7. $\log_a(6x^2 + 11) - \log_a x = \log_a \left[\frac{6x^2 + 11}{x} \right]$ (subtraction law) B1
- $2 \log_a 5 = \log_a 5^2$ (power law) B1
- $\frac{6x^2 + 11}{x} = 5^2$ (removing logs) M1
- An attempt to solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$,
with $a \times c = 6$ and $b \times d = 11$ m1
- $(2x - 1)(3x - 11) = 0 \Rightarrow x = \frac{1}{2}, \frac{11}{3}$ (both values, c.a.o.) A1
- Note: Answer only with no working earns 0 marks**
8. (a) (i) $A(1, -3)$ B1
- (ii) Gradient $AP = \frac{\text{inc in } y}{\text{inc in } x}$ M1
- Gradient $AP = \frac{(-7) - (-3)}{4 - 1} = -\frac{4}{3}$
- (f.t. candidate's coordinates for A) A1
- Use of $m_{\text{tan}} \times m_{\text{rad}} = -1$ M1
- Equation of tangent is:
 $y - (-7) = \frac{3}{4}(x - 4)$ (f.t. candidate's gradient for AP) A1
- (b) An attempt to substitute $(x + 4)$ for y in the equation of the circle and form quadratic in x M1
- $x^2 + (x + 4)^2 - 2x + 6(x + 4) - 15 = 0 \Rightarrow 2x^2 + 12x + 25 = 0$ A1
- An attempt to calculate value of discriminant m1
- Discriminant $= 144 - 200 < 0 \Rightarrow$ no points of intersection
(f.t. one slip) A1
9. (a) $4\theta = 5.2$ M1
- $\theta = 1.3$ A1
- (b) $RP = 4 \times \tan 1.3 \text{ cm}$ (o.e.) (f.t. candidate's value for θ) B1
- Area of triangle $POR = \frac{1}{2} \times 4 \times 4 \times \tan 1.3 \text{ cm}^2$ (o.e.)
- (f.t. candidate's value for θ) M1
- Area of sector $POQ = \frac{1}{2} \times 4 \times 4 \times 1.3 \text{ cm}^2$
- (f.t. candidate's value for θ) M1
- Either: Area of triangle $POR = 28.8 \text{ cm}^2$
- Or: Area of sector $POQ = 10.4 \text{ cm}^2$
- (f.t. candidate's value for θ) A1
- An attempt to find shaded area by subtracting the derived area of the sector from the derived area of the triangle M1
- Shaded area $= 28.8 - 10.4 = 18.4 \text{ cm}^2$ (c.a.o.) A1

C3

- 1.
- | | | | |
|--|--|-------------|----------------------------|
| | 4 | 1 | |
| | 4.5 | 1.138071187 | |
| | 5 | 1.309016994 | |
| | 5.5 | 1.527202251 | (5 values correct) B2 |
| | 6 | 1.816496581 | (3 or 4 values correct) B1 |
| | Correct formula with $h = 0.5$ | | M1 |
| | $I \approx \frac{0.5}{3} \times \{1 + 1.816496581 + 4(1.138071187 + 1.527202251) + 2(1.309016994)\}$ | | |
| | $I \approx 16.09562432 \times 0.5 \div 3$ | | |
| | $I \approx 2.682604054$ | | |
| | $I \approx 2.683$ | | (f.t. one slip) A1 |

Note: Answer only with no working earns 0 marks

2. (a) e.g. $\theta = \frac{\pi}{4}$
- | | | |
|--|---|----|
| $\sec^2 \theta = 2$ | (choice of θ and one correct evaluation) | B1 |
| $1 - \operatorname{cosec}^2 \theta = -1$ | (both evaluations correct but different) | B1 |
- (b) $3(1 + \cot^2 \theta) = 11 - 2 \cot \theta$. (correct use of $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$) M1
- An attempt to collect terms, form and solve quadratic equation in $\cot \theta$, either by using the quadratic formula or by getting the expression into the form $(a \cot \theta + b)(c \cot \theta + d)$, with $a \times c = \text{coefficient of } \cot^2 \theta$ and $b \times d = \text{candidate's constant}$ m1
- $3 \cot^2 \theta + 2 \cot \theta - 8 = 0 \Rightarrow (3 \cot \theta - 4)(\cot \theta + 2) = 0$
- $\Rightarrow \cot \theta = \frac{4}{3}, \cot \theta = -2$
- $\Rightarrow \tan \theta = \frac{3}{4}, \tan \theta = -\frac{1}{2}$ (c.a.o.) A1
- $\theta = 36.87^\circ, 216.87^\circ$ B1
- $\theta = 153.43^\circ, 333.43^\circ$ B1 B1
- Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
- $\tan \theta = +, -, \text{ f.t. for 3 marks, } \tan \theta = -, -, \text{ f.t. for 2 marks}$
- $\tan \theta = +, +, \text{ f.t. for 1 mark}$

3. (a) $\frac{d(2y^2)}{dx} = 4y \frac{dy}{dx}$ B1
 $\frac{d(3x^2y)}{dx} = 3x^2 \frac{dy}{dx} + 6xy$ B1
 $\frac{d(x^4)}{dx} = 4x^3, \frac{d(15)}{dx} = 0$ B1
 $\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$ (c.a.o.) B1
- (b) (i) Differentiating $\ln t$ and $t^3 - 7t$ with respect to t , at least one correct M1
candidate's x -derivative $= \frac{1}{t}$,
candidate's y -derivative $= 3t^2 - 7$ (both values) A1
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = 3t^3 - 7t$ (c.a.o.) A1
- (ii) $\frac{d}{dt} \left[\frac{dy}{dx} \right] = 9t^2 - 7$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1
Use of $\frac{d^2y}{dx^2} = \frac{d}{dt} \left[\frac{dy}{dx} \right] \div \text{candidate's } x\text{-derivative}$ M1
 $\frac{d^2y}{dx^2} = 9t^3 - 7t$ (f.t. one slip) A1
When $t = \frac{1}{3}, \frac{d^2y}{dx^2} = -2$ (c.a.o.) A1
4. $x_0 = 0.4$
 $x_1 = 0.406628571$ (x_1 correct, at least 4 places after the point) B1
 $x_2 = 0.405137517$
 $x_3 = 0.405479348$
 $x_4 = 0.405401314 = 0.4054$ (x_4 correct to 4 decimal places) B1
An attempt to check values or signs of $f(x)$ at $x = 0.40535, x = 0.40545$ M1
 $f(0.40535) = -5.66 \times 10^{-4} < 0, f(0.40545) = 2.94 \times 10^{-4} > 0$ A1
Change of sign $\Rightarrow \alpha = 0.4054$ correct to four decimal places A1

5.	(a)	(i)	$\frac{dy}{dx} = \frac{1}{2} \times (2 + 5x^3)^{-1/2} \times f(x)$	$(f(x) \neq 1)$	M1
			$\frac{dy}{dx} = \frac{15x^2}{2\sqrt{2 + 5x^3}}$		A1
		(ii)	$\frac{dy}{dx} = x^2 \times f(x) + \sin 3x \times g(x)$	$(f(x) \neq 1, g(x) \neq 1)$	M1
			$\frac{dy}{dx} = x^2 \times f(x) + \sin 3x \times g(x)$		
				(either $f(x) = 3 \cos 3x$ or $g(x) = 2x$)	A1
			$\frac{dy}{dx} = x^2 \times 3 \cos 3x + \sin 3x \times 2x$	(all correct)	A1
		(iii)	$\frac{dy}{dx} = \frac{x^4 \times m \times e^{2x} - e^{2x} \times 4x^3}{(x^4)^2}$	$(m = 1, 2)$	M1
			$\frac{dy}{dx} = \frac{x^4 \times 2 \times e^{2x} - e^{2x} \times 4x^3}{(x^4)^2}$		A1
			$\frac{dy}{dx} = \frac{2e^{2x}(x - 2)}{x^5}$		A1
		(b)	$x = \tan y \Rightarrow \frac{dx}{dy} = \sec^2 y$		B1
			Appropriate use of $\sec^2 y = 1 + \tan^2 y$		M1
			Appropriate use of $1 + \tan^2 y = 1 + x^2$		m1
			$\frac{dy}{dx} = \frac{1}{1 + x^2}$	(c.a.o.)	A1

6. (a) (i) $\int \cos 4x \, dx = k \times \sin 4x + c$ ($k = 1, 4, \pm \frac{1}{4}$) M1
 $\int \cos 4x \, dx = \frac{1}{4} \times \sin 4x + c$ A1
- (ii) $\int 5e^{2-3x} \, dx = k \times 5e^{2-3x} + c$ ($k = 1, -3, \pm \frac{1}{3}$) M1
 $\int 5e^{2-3x} \, dx = -\frac{1}{3} \times 5e^{2-3x} + c$ A1
- (iii) $\int \frac{3}{(6x-7)^5} \, dx = -\frac{3}{4k} \times (6x-7)^{-4} + c$ ($k = 1, 6, \frac{1}{6}$) M1
 $\int \frac{3}{(6x-7)^5} \, dx = -\frac{3}{24} \times (6x-7)^{-4} + c$ A1

Note: The omission of the constant of integration is only penalised once.

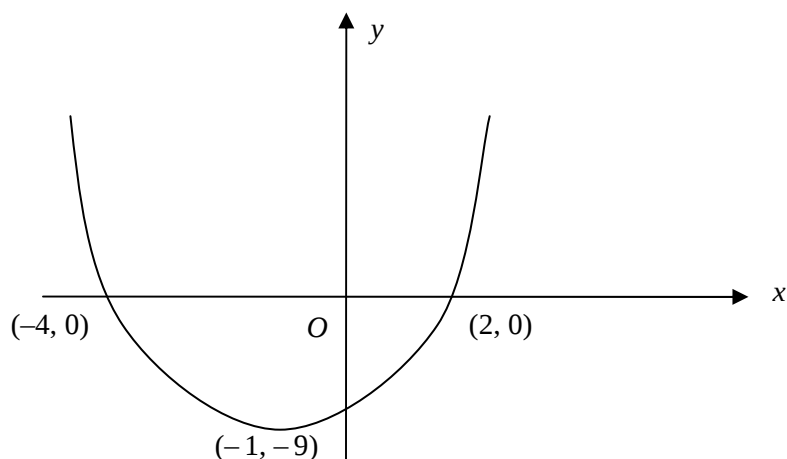
- (b) $\int \frac{9}{2x+5} \, dx = (9) \times k \times \ln |2x+5|$ ($k = 1, 2, \frac{1}{2}$) M1
 $\int \frac{9}{2x+5} \, dx = \left[9 \times \frac{1}{2} \times \ln |2x+5| \right]$ A1
A correct method for substitution of limits in an expression of the form $m \times \ln |2x+5|$ M1
 $\int_1^4 \frac{9}{2x+5} \, dx = \frac{9}{2} \times \ln \left(\frac{13}{7} \right) = 2.786$ (c.a.o.) A1

7. (a) $8|x| = 6$ B1
 $x = \pm \frac{3}{4}$
(f.t. candidate's $a|x| = b$, with at least one of a, b correct) B1
- (b) Trying to solve either $3x - 1 > 5$ or $3x - 1 < -5$ M1
 $3x - 1 > 5 \Rightarrow x > 2$
 $3x - 1 < -5 \Rightarrow x < -\frac{4}{3}$ (both inequalities) A1
Required range: $x < -\frac{4}{3}$ or $x > 2$ (f.t. one slip) A1

Alternative mark scheme

- $(3x - 1)^2 > 25$ (forming and trying to solve quadratic) M1
Critical values $x = -\frac{4}{3}$ and $x = 2$ A1
Required range: $x < -\frac{4}{3}$ or $x > 2$ (f.t. one slip in critical values) A1

8.



Concave up curve and y-coordinate of minimum = -9

B1

x-coordinate of minimum = -1

B1

Both points of intersection with x-axis

B1

9. (a) $R(f) = [1, \infty)$

B1

(b) $y = 4x^2 - 3$ and an attempt to isolate x

M1

$$4x^2 = y + 3 \Rightarrow x = (\pm) \frac{1}{2}\sqrt{y + 3}$$

A1

$$x = -\frac{1}{2}\sqrt{y + 3}$$

(f.t. one slip)

A1

$$f^{-1}(x) = -\frac{1}{2}\sqrt{x + 3}$$

(f.t. candidate's expression for x)

A1

$$R(f^{-1}) = (-\infty, -1], D(f^{-1}) = [1, \infty)$$

(both intervals, f.t. candidate's $R(f)$)

B1

(c) (i) $f^{-1}(6) = -3/2$ (f.t. only for omitted - sign in candidate's correct expression for $f^{-1}(x)$)

B1

(ii) Evaluation of $f(k)$, where k is candidate's value for $f^{-1}(6)$

M1

$$f(-3/2) = 6$$

(c.a.o.)

A1

10. (a) $gf(x) = 4[f(x)]^3 + 7$

M1

$$gf(x) = 4(e^x)^3 + 7 = 4e^{3x} + 7$$

A1

(b) $D(gf) = [0, \infty)$

B1

$$R(gf) = [11, \infty)$$

B1

(c) (i) $gf(x) = 18 \Rightarrow 4e^{3x} + 7 = 18 \Rightarrow e^{3x} = 11/4 \Rightarrow 3x = \ln(11/4)$

(f.t. candidate's gf and allow one algebraic slip)

M1

$$x = 0.337$$

(c.a.o.)

A1

(ii) Either: e.g. $k = 9$ since $9 \notin R(gf)$ (f.t. candidate's $R(gf)$)

Or: Verification that candidate's choice of k does not yield a value of $x \in D(gf)$

B1

FP1

1.
$$S_n = \sum_{r=1}^n r^2(2r+1)$$
 M1

$$= 2\sum_{r=1}^n r^3 + \sum_{r=1}^n r^2$$
 A1

$$= \frac{2n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$$
 A1

$$= \frac{n(n+1)}{12}(6n^2 + 6n + 4n + 2)$$
 m1
 [m1 attempt to factorise]

$$= \frac{n(n+1)(3n^2 + 5n + 1)}{6}$$
 cao A1

2. (a) Using row operations, M1

$$x + 2y + z = 1$$

$$y + z = -1$$
 A1

$$2y + 2z = 3 - \lambda$$
 A1
 The third line is twice the second line so

$$3 - \lambda = -2, \lambda = 5$$
 A1
 (b) Put $z = \alpha$. M1
 Then $y = -1 - \alpha, x = \alpha + 3$ A1A1

3. (a) $(2+i)(-1+i) = -2 - 1 - i + 2i$ M1

$$= -3 + i$$
 A1

$$\frac{1}{z} = -3 + i + 4(1-i)$$
 M1

$$= 1 - 3i$$
 A1

$$z = \frac{(1+3i)}{(1-3i)(1+3i)}$$
 M1
 [M1 inverting and attempting to rationalise]

$$= \frac{1}{10} + \frac{3}{10}i$$
 A1
 (b) $\text{Mod}(z) = \frac{\sqrt{10}}{10} (0.316), \text{Arg}(z) = 71.6^\circ \quad (1.25 \text{ rad})$ B1B1
 [FT on their z]

4. (a) $\alpha + \beta + \gamma = 3, \beta\gamma + \gamma\alpha + \alpha\beta = 2, \alpha\beta\gamma = -4$ B2
[B1 for 2 correct]

Consider

$$\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma} = \frac{\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2}{\alpha\beta\gamma}$$
 M1

[M1 attempting to put over common denominator]

$$= \frac{(\beta\gamma + \gamma\alpha + \alpha\beta)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)}{\alpha\beta\gamma}$$
 A1

$$= \frac{2^2 - 2 \times -4 \times 3}{-4} = -7$$
 A1

(b) $\frac{\gamma\alpha}{\beta} \times \frac{\alpha\beta}{\gamma} + \frac{\alpha\beta}{\gamma} \times \frac{\beta\gamma}{\alpha} + \frac{\beta\gamma}{\alpha} \times \frac{\gamma\alpha}{\beta} = \alpha^2 + \beta^2 + \gamma^2$ M1

[M1 for considering sum of products]

$$= (\alpha + \beta + \gamma)^2 - 2(\beta\gamma + \gamma\alpha + \alpha\beta)$$
 A1

$$= 9 - 2 \times 2 = 5$$
 A1

$$\frac{\beta\gamma}{\alpha} \times \frac{\gamma\alpha}{\beta} \times \frac{\alpha\beta}{\gamma} = \alpha\beta\gamma = -4$$
 M1A1

The required equation is $x^3 + 7x^2 + 5x + 4 = 0$ B1
[FT their previous work]

5. (a) The statement is true for $n = 1$ since the formula gives

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$
 B1

Let the statement be true for $n = k$, ie

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^k = \begin{bmatrix} 1 & 2^k - 1 \\ 0 & 2^k \end{bmatrix}$$
 M1

Consider

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^k \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$
 M1

$$= \begin{bmatrix} 1 & 2^k - 1 \\ 0 & 2^k \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$
 A1

$$= \begin{bmatrix} 1 & 1 + 2(2^k - 1) \\ 0 & 2 \cdot 2^k \end{bmatrix}$$
 A1

$$= \begin{bmatrix} 1 & 2^{k+1} - 1 \\ 0 & 2^{k+1} \end{bmatrix}$$
 A1

So true for $n = k \Rightarrow$ true for $n = k + 1$, and since true for $n = 1$ the result is proved by induction. A1

[Only award final A1 if correctly presented throughout]

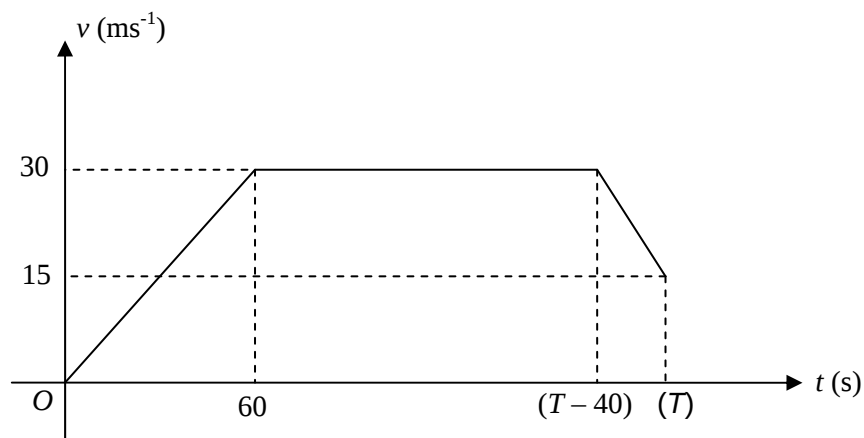
6. (a) (i) Determinant = $1(\lambda + 2) - 2(\lambda^2 + 4) + 3(\lambda - 2)$ M1
 $= -2\lambda^2 + 4\lambda - 12$ A1
- (ii) The condition for this never to be zero is ' $\Delta = b^2 - 4ac < 0$ ' M1
 Here, $\Delta = 16 - 96 < 0$ A1
 [FT their expression in (a)(i) if possible]
- (b) (i) When $\lambda = 1$,
- $$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -2 \\ 2 & 1 & 1 \end{bmatrix}$$
- Cofactor matrix = $\begin{bmatrix} 3 & -5 & -1 \\ 1 & -5 & 3 \\ -7 & 5 & -1 \end{bmatrix}$ M1A1
- [Award M1 if at least three terms correct]
- Adjugate matrix = $\begin{bmatrix} 3 & 1 & -7 \\ -5 & -5 & 5 \\ -1 & 3 & -1 \end{bmatrix}$ A1
- [FT on cofactor matrix]
- Determinant = -10 cao B1
- $$\mathbf{A}^{-1} = -\frac{1}{10} \begin{bmatrix} 3 & 1 & -7 \\ -5 & -5 & 5 \\ -1 & 3 & -1 \end{bmatrix}$$
- A1
- [FT on previous work]
- (ii) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{10} \begin{bmatrix} 3 & 1 & -7 \\ -5 & -5 & 5 \\ -1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}$ M1
- $$= \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$
- A1
- [FT on their inverse matrix]

7. (a) $\ln f(x) = x \ln 2 + \frac{1}{x} \ln 3$ B1
- $$\frac{f'(x)}{f(x)} = \ln 2 - \frac{1}{x^2} \ln 3$$
- B1B1
- $$f'(x) = 2^x \times 3^{1/x} \left(\ln 2 - \frac{1}{x^2} \ln 3 \right)$$
- B1
- (b) At a stationary point, $f'(x) = 0$ so M1
- $$\ln 2 = \frac{1}{x^2} \ln 3$$
- $$x = \sqrt{\ln 3 / \ln 2} = 1.25...$$
- A1
- Stat value = 5.73 A1
- It is a minimum because $f'(x) < 0$ for any value of $x < 1.25$ and $f'(x) > 0$ for any value of $x > 1.25$ [Accept specific values] A1
8. (a) Reflection matrix in $y - x = 0 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1
- Translation matrix = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ B1
- Reflection matrix in $y + x = 0 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1
- $$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
- M1
- $$= \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$
- A1
- $$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$
- (b) Fixed points satisfy
- $$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
- M1
- $$-x + 1 = x$$
- $$-y - 2 = y$$
- A1
- $$(x, y) = (1/2, -1)$$
- A1

9. (a) $u + iv = (x + iy)^2$ M1
 $= x^2 - y^2 + 2ixy$ A1
 $u = x^2 - y^2, v = 2xy$ A1
- (b) Putting $y = x^2$, or $x = \sqrt{y}$ M1
[M1 attempting to eliminate x or y]
 $u = x^2 - x^4, v = 2x^3$ $u = y - y^2, v = 2y^{3/2}$ A1
Eliminating x or y ,
 $u = \left(\frac{v}{2}\right)^{2/3} - \left(\frac{v}{2}\right)^{4/3}$ A1
- (c) (i) $\alpha = 8^{2/3} - 8^{4/3} = -12$ B1
- (ii) It now follows that
 $2x^3 = 16$ so $x = 2$ M1A1
 $y = x^2 = 4$ A1
[The required point is (2,4)]

M1

1. (a)



v - t graph and $(0, 0)$ to $(60, 30)$ M1

$(60, 30)$ to $(?, 30)$ A1

$(?, 30)$ to $(?, 15)$ A1

All labels and units; all correct A1

(b) $\text{acceleration} = \frac{30 - 0}{60}$ M1

$= \underline{0.5 \text{ ms}^{-1}}$ A1

$\text{distance} = 0.5 \times 60 \times 30$ any correct method M1

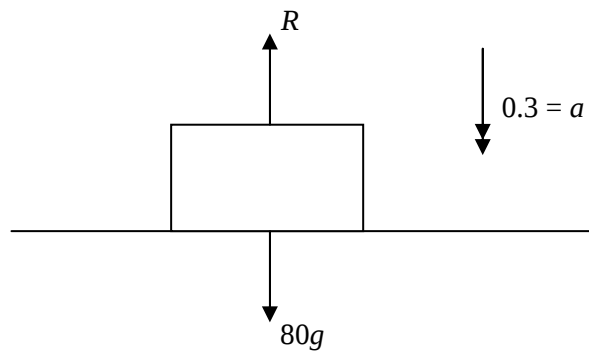
$= \underline{900 \text{ m}}$ A1

(c) $\text{Total area under graph} = 24 \times 1000$ M1

$900 + (T - 40 - 60) \times 30 + 0.5 \times 40 (30 + 15) = 24000$ A1 A1

$\text{Total time} = T = \underline{840 \text{ s}}$ A1

2. (a)

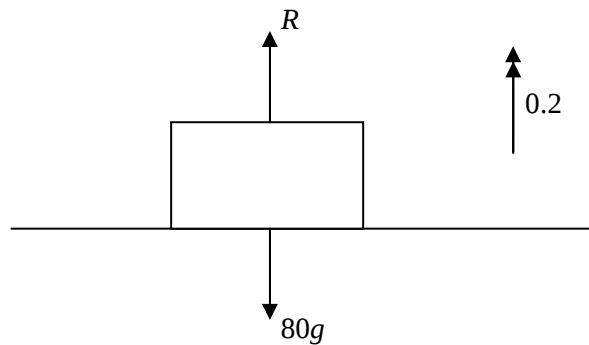


N2L applied to trunk dim. correct, R and $80g$ opposing M1

$$80g - R = 80 \times 0.3 \quad \text{A1}$$

$$R = \underline{760 \text{ N}} \quad \text{cao} \quad \text{A1}$$

(b)



N2L applied to trunk dim. correct, R and $80g$ opposing M1

$$R - 80g = 80 \times 0.2 \quad \text{A1}$$

$$R = \underline{800 \text{ N}} \quad \text{cao} \quad \text{A1}$$

(c) $R = 80g$ since $a = 0$

$$= \underline{784 \text{ N}} \quad \text{B1}$$

3. (a) Using $v = u + at$ with $u = 0$, $t = 0.8$, $a = (+) 9.8$ (downwards positive) M1

$$v = 0 + 9.8 \times 0.8 \quad \text{A1}$$

$$v = \underline{7.84 \text{ ms}^{-1}} \quad \text{A1}$$

(b) Using $v^2 = u^2 + 2as$ with $u = u$, $s = 0.9$, $v = 0$ (upwards positive) M1

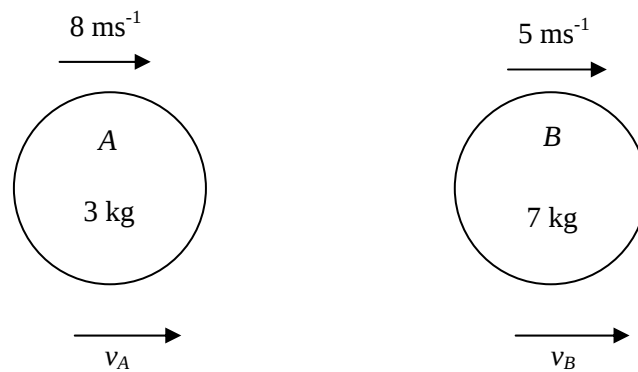
$$0 = u^2 - 2 \times 9.8 \times 0.9 \quad \text{A1}$$

$$u = \underline{4.2 \text{ ms}^{-1}} \quad \text{A1}$$

$$\text{Coefficient of restitution} = \frac{4.2}{7.84} = \left(\frac{15}{28} \right) \quad \text{M1}$$

$$= \underline{0.536} \text{ (to 3 sig figs)} \quad \text{ft } u, v \quad \text{A1}$$

4. (a)



Conservation of momentum M1

$$3 \times 8 + 7 \times 5 = 3v_A + 7v_B \quad \text{A1}$$

Restitution M1

$$v_B - v_A = -0.4(5 - 8) \quad \text{A1}$$

$$-7 v_A + 7v_B = 8.4$$

$$3 v_A + 7v_B = 59$$

Subtract $10 v_A = 50.6$ m1

$$v_A = \underline{5.06 \text{ ms}^{-1}} \quad \text{cao} \quad \text{A1}$$

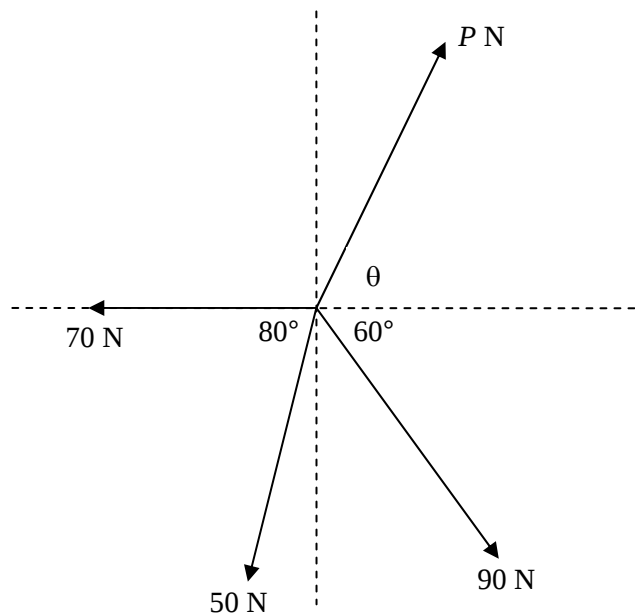
$$v_B = \underline{6.26 \text{ ms}^{-1}} \quad \text{cao} \quad \text{A1}$$

(b) Impulse required = change in momentum of B used M1

$$= 7(6.26 - 5)$$

$$= \underline{8.82 \text{ Ns}} \quad \text{ft } v_B > 5 \quad \text{A1}$$

5.



Resolve in direction parallel to 70 N ($\rightarrow$) all forces M1

$$P\cos\theta + 90\cos 60^\circ = 70 + 50\cos 80^\circ \quad \text{A1}$$

$$P\cos\theta = 33.6824$$

Resolve in direction perpendicular to 70 N ($\uparrow$) all forces M1

$$P\sin\theta = 90\sin 60^\circ + 50\sin 80^\circ \quad \text{A1}$$

$$P\sin\theta = 127.1827$$

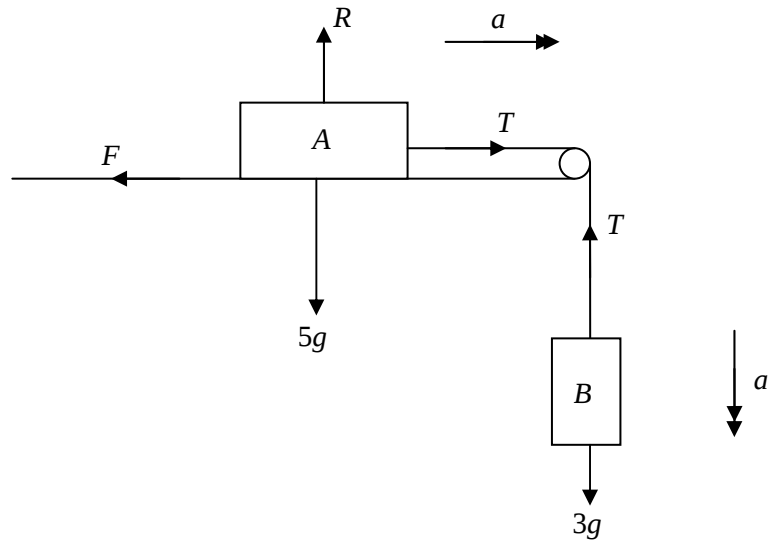
$$P = \sqrt{33.6824^2 + 127.1827^2} \quad \text{M1}$$

$$P = \underline{131.6 \text{ N}} \quad \text{ft} \quad \text{A1}$$

$$\theta = \tan^{-1}\left(\frac{127.1827}{33.6824}\right) \quad \text{M1}$$

$$\theta = \underline{75.2^\circ} \quad \text{ft} \quad \text{A1}$$

6. (a)



At A, resolve vertically $R = 5g$ si B1

Limiting friction $= \mu R = 0.4 \times 5g$ si B1

$$F = 19.6 \text{ N}$$

N2L applied to B M1

$$3g - T = 3a \quad \text{A1}$$

N2L applied to A M1

$$T - F = 5a \quad \text{ft } F \quad \text{A1}$$

Adding $8a = 3 \times 9.8 - 19.6$ m1

$$a = \underline{1.225 \text{ ms}^{-2}} \quad \text{cao} \quad \text{A1}$$

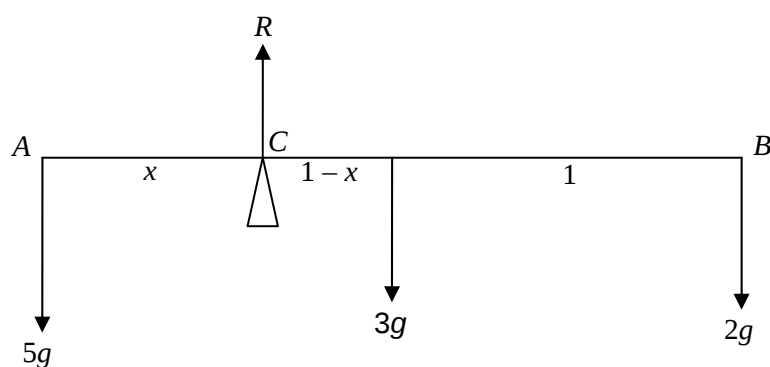
$$T = \underline{25.725 \text{ N}} \quad \text{cao} \quad \text{A1}$$

(b) Least value of μ is given by $a = 0$ M1

$$3g - 5\mu g = 0 \quad \text{m1}$$

$$\text{least } \mu = \underline{0.6} \quad \text{cao} \quad \text{A1}$$

7.



Resolve vertically M1

$$R = 5g + 3g + 2g$$

$$= 10g$$

$$= \underline{98 \text{ N}} \quad \text{A1}$$

Moments about C all forces M1

$$5gx = 3g(1-x) + 2g(2-x) \quad \text{B1 A1}$$

$$5x = 3 - 3x + 4 - 2x$$

$$10x = 7$$

$$x = \underline{0.7} \quad \text{A1}$$

8.	(a)	Area	from AD	from AB	
		$ABCD$	120	5	6 B1
		Circle	9π	4	7 B1
		Lamina	$120 - 9\pi$	x	y B1
		Moments from AD			M1
		$120 \times 5 = 9\pi \times 4 + (120 - 9\pi) \times x$			A1
		$x = \underline{5.308 \text{ cm}}$			cao A1
		Moments from AB			M1
		$120 \times 6 = 9\pi \times 7 + (120 - 9\pi) \times y$			A1
		$y = \underline{5.692 \text{ cm}}$			cao A1
	(b)	Required angle $= \theta = \tan^{-1}\left(\frac{y}{x}\right)$			M1
		$\theta = \tan^{-1}\left(\frac{5.692}{5.308}\right)$			ft x, y A1
		$\theta = \underline{47.0^\circ}$			ft x, y A1
	(c)	$DP = \underline{5.308 \text{ cm}}$			ft x B1

S1

1. (a) (1,1) (1,2) (1,3) (1,4)
(2,1) (2,2) (2,3) (2,4)
(3,1) (3,2) (3,3) (3,4)
(4,1) (4,2) (4,3) (4,4) M1A1
- (b) (i) Attempting to count the number of pairs. M1

$$\text{Prob} = \frac{3}{16}$$
 A1
 (ii) Attempting to count the number of pairs M1

$$\text{Prob} = \frac{6}{16}$$
 A1
2. (a) $p + p = 0.64$ M1
 $p = 0.32$ A1
- (b) (i) $P(A \cap B) = p^2$ B1
 Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $0.64 = 2p - p^2$ M1
 whence $25p^2 - 50p + 16 = 0$ (so $k = 16$) A1
- (ii) $p = \frac{50 \pm \sqrt{2500 - 1600}}{50}$ M1
 $p = 0.4$ cao A1
 [Award A0 if both 0.4 and 1.6 are given]
 Special case : If the solutions to (a) and (b) are interchanged, mark according to the scheme and then deduct 3 marks subject to the final mark being non-negative.
- 3 (a) $\text{Prob} = \frac{6}{12} \times \frac{4}{11} \times \frac{2}{10} \times 6$ or $\binom{6}{1} \times \binom{4}{1} \times \binom{2}{1} \div \binom{12}{3}$ M1A1
 [M1 multiplying sensible probabilities, A1 the 6]
 $= \frac{12}{55}$ cao A1
- (b) $\text{Prob} = \frac{6}{12} \times \frac{5}{11} \times \frac{4}{10}$ or $\binom{6}{3} \div \binom{12}{3} = \frac{1}{11}$ M1A1
- (c) $P(\text{All pop}) = \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$ or $\binom{4}{3} \div \binom{12}{3} \left(\frac{1}{55} \right)$ B1
 $P(\text{All same type}) = \text{sum of (b) and (c)} = \frac{6}{55}$ cao M1A1

4. Mean = $0.2n$ and SD = $\sqrt{0.8 \times 0.2n}$ B1B1
 We are given that

$$0.2n = 2\sqrt{0.8 \times 0.2n}$$
 M1

$$0.04n^2 = 0.64n$$
 A1

$$n = 16$$
 A1
5. (a) (i) Prob = $e^{-15} \times \frac{15^8}{8!}$ or $0.0374 - 0.0180$ or $0.9820 - 0.9626 = 0.0194$
 [M0 answer only] M1A1
- (ii) Prob = $0.9170 - 0.0699$ or $0.9301 - 0.0830$ B1B1
 $= 0.8471$ cao B1
 [No marks answer only]
- (b) (i) $Y = 8X - 50$ B1
- (ii) $E(Y) = 8 \times 15 - 50 = 70$ M1A1
 $\text{Var}(Y) = 64 \times 15 = 960$ M1A1
 [FT on (i)]
6. (a) P(found guilty) = $0.8 \times 0.9 + 0.2 \times 0.05$ M1A1
 $= 0.73$ A1
- (b) Reqd prob = $\frac{0.8 \times 0.9}{0.73}$ B1B1
 $= \frac{72}{73}$ cao B1
 [FT the denominator from (a)]
- 7 (a) (i) $[0, 0.4]$ [Accept ()] B1
- (ii) $E(X) = 0.4 - \alpha + 2.2\alpha + 3(0.6 - \alpha)$
 $= 2.2$ (so independent of α) A1
- (iii) $E(X^2) = 0.4 - \alpha + 4.2\alpha + 9(0.6 - \alpha)$ M1
 $= 5.8 - 2\alpha$ A1
 $\text{Var}(X) = 5.8 - 2\alpha - 2.2^2$ A1
 $\text{Var}(X) = 0.66$ gives $\alpha = 0.15$ A1
 [FT their $E(X)$ where possible]
- (b) Possibilities are 1,1 ; 2,2 ; 3,3 B1
 Prob = $(0.15^2 + 0.5^2 + 0.35^2)$ M1A1
 $= 0.395$ cao A1

8. (a) (i) B(20,0.05) B1
[Parameters may be given later]
- (ii) $\text{Prob} = \binom{20}{1} \times 0.05^1 \times 0.95^{19} = 0.377$ M1A1
- (iii) $P(X \geq 3) = 1 - 0.9245 = 0.0755$ M1A1
- (b) The number broken, Y , is approx Poi(10). B1
 $P(Y < 5) = 1 - 0.9707 = 0.0293$ M1A1
9. (a) $E(X) = \int_1^3 \frac{1}{6}x(x+1) dx$ M1A1
[M1 for integral of $xf(x)$, A1 completely correct, limits may appear on next line]
- $$= \left[\frac{x^3}{18} + \frac{x^2}{12} \right]_1^3$$
- A1
- $$= \frac{19}{9} \quad (2.11) \quad \text{cao}$$
- A1
- (b) (i) $F(x) = \int_1^x \frac{1}{6}(t+1) dt$ M1
[Award M1 for integral of $f(x)$]
- $$= \left[\frac{t^2}{12} + \frac{t}{6} \right]_1^x$$
- A1
- $$= \frac{x^2}{12} + \frac{x}{6} - \frac{1}{4} \quad \left\{ \frac{1}{12}(x^2 + 2x - 3) \right\}$$
- A1
- (ii) $F(4) = 1$ B1
- (iii) $\text{Prob} = F(2) - F(1.5)$ M1

$$= \frac{2^2}{12} + \frac{2}{6} - \frac{1}{4} - \frac{1.5^2}{12} - \frac{1.5}{6} + \frac{1}{4}$$

A1

$$= \frac{11}{48} \quad (0.229)$$

A1

[FT on their $F(x)$]

(iv) The median m satisfies

$$\frac{m^2}{12} + \frac{m}{6} - \frac{1}{4} = 0.5$$

M1

[M1 for putting their $F(x) = 0.5$]

$$m = \frac{-2 \pm \sqrt{4 + 36}}{2}$$

m1

[Only award if a quadratic equation is being solved]

$$m = 2.16 \quad (\sqrt{10} - 1) \quad \text{cao}$$

A1



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