C3

1.

4	1		
4·5	1·138071187		
5	1·309016994		
5·5	1·527202251	(5 values correct)	B2
6	1·816496581	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{3} \times \{1 + 1{\cdot}816496581 + 4(1{\cdot}138071187 + 1{\cdot}527202251) + 2(1{\cdot}309016994)\}$

$I \approx 16{\cdot}09562432 \times 0{\cdot}5 \div 3$

$I \approx 2{\cdot}682604054$

$I \approx 2{\cdot}683$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) e.g. $\theta = \frac{\pi}{4}$

$\sec^2\theta = 2$ (choice of θ and one correct evaluation) B1

$1 - \operatorname{cosec}^2\theta = -1$ (both evaluations correct but different) B1

(*b*) $3(1 + \cot^2\theta) = 11 - 2\cot\theta$. (correct use of $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant m1

$3\cot^2\theta + 2\cot\theta - 8 = 0 \Rightarrow (3\cot\theta - 4)(\cot\theta + 2) = 0$

$\Rightarrow \cot\theta = \frac{4}{3}, \cot\theta = -2$

$\Rightarrow \tan\theta = \frac{3}{4}, \tan\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 36{\cdot}87°, 216{\cdot}87°$ B1

$\theta = 153{\cdot}43°, 333{\cdot}43°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\tan\theta$ = +, –, f.t. for 3 marks, $\tan\theta$ = –, –, f.t. for 2 marks

$\tan\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(2y^2) = 4y\frac{dy}{dx}$ B1

$\frac{d}{dx}(3x^2y) = 3x^2\frac{dy}{dx} + 6xy$ B1

$\frac{d}{dx}(x^4) = 4x^3$, $\frac{d}{dx}(15) = 0$ B1

$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$ (c.a.o.) B1

(*b*) (i) Differentiating ln t and $t^3 - 7t$ with respect to t, at least one correct M1

candidate's x-derivative = $\frac{1}{t}$,

candidate's y-derivative = $3t^2 - 7$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = 3t^3 - 7t$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 9t^2 - 7$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = 9t^3 - 7t$ (f.t. one slip) A1

When $t = \frac{1}{3}$, $\frac{d^2y}{dx^2} = -2$ (c.a.o.) A1

4. $x_0 = 0{\cdot}4$

$x_1 = 0{\cdot}406628571$ (x_1 correct, at least 4 places after the point) B1

$x_2 = 0{\cdot}405137517$

$x_3 = 0{\cdot}405479348$

$x_4 = 0{\cdot}405401314 = 0{\cdot}4054$ (x_4 correct to 4 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 0{\cdot}40535$, $x = 0{\cdot}40545$ M1

$f(0{\cdot}40535) = -5{\cdot}66 \times 10^{-4} < 0$, $f(0{\cdot}40545) = 2{\cdot}94 \times 10^{-4} > 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}4054$ correct to four decimal places A1

5. (*a*) (i) $\frac{dy}{dx} = \frac{1}{2} \times (2 + 5x^3)^{-1/2} \times f(x)$ ($f(x) \neq 1$) M1

$\frac{dy}{dx} = \frac{15x^2}{2\sqrt{2 + 5x^3}}$ A1

(ii) $\frac{dy}{dx} = x^2 \times f(x) + \sin 3x \times g(x)$ ($f(x) \neq 1$, $g(x) \neq 1$) M1

$\frac{dy}{dx} = x^2 \times f(x) + \sin 3x \times g(x)$ (either $f(x) = 3\cos 3x$ or $g(x) = 2x$) A1

$\frac{dy}{dx} = x^2 \times 3\cos 3x + \sin 3x \times 2x$ (all correct) A1

(iii) $\frac{dy}{dx} = \frac{x^4 \times m \times e^{2x} - e^{2x} \times 4x^3}{(x^4)^2}$ ($m = 1, 2$) M1

$\frac{dy}{dx} = \frac{x^4 \times 2 \times e^{2x} - e^{2x} \times 4x^3}{(x^4)^2}$ A1

$\frac{dy}{dx} = \frac{2e^{2x}(x - 2)}{x^5}$ A1

(*b*) $x = \tan y \Rightarrow \frac{dx}{dy} = \sec^2 y$ B1

Appropriate use of $\sec^2 y = 1 + \tan^2 y$ M1

Appropriate use of $1 + \tan^2 y = 1 + x^2$ m1

$\frac{dy}{dx} = \frac{1}{1 + x^2}$ (c.a.o.) A1

6. (*a*) (i) $\int \cos 4x \, dx = k \times \sin 4x + c$ ($k = 1, 4, \pm {}^1/_4$) M1

$\int \cos 4x \, dx = {}^1/_4 \times \sin 4x + c$ A1

(ii) $\int 5e^{2-3x} dx = k \times 5e^{2-3x} + c$ ($k = 1, -3, \pm {}^1/_3$) M1

$\int 5e^{2-3x} dx = -{}^1/_3 \times 5e^{2-3x} + c$ A1

(iii) $\int \frac{3}{(6x-7)^5} dx = -\frac{3}{4k} \times (6x-7)^{-4} + c$ ($k = 1, 6, {}^1/_6$) M1

$\int \frac{3}{(6x-7)^5} dx = -\frac{3}{24} \times (6x-7)^{-4} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \frac{9}{2x+5} dx = (9) \times k \times \ln|2x+5|$ ($k = 1, 2, {}^1/_2$) M1

$\int \frac{9}{2x+5} dx = \left[9 \times {}^1/_2 \times \ln|2x+5| \right]$ A1

A correct method for substitution of limits in an expression of the form $m \times \ln|2x+5|$ M1

$\int_1^4 \frac{9}{2x+5} dx = \frac{9}{2} \times \ln({}^{13}/_7) = 2{\cdot}786$ (c.a.o.) A1

7. (*a*) $8|x| = 6$ B1

$x = \pm {}^3/_4$

(f.t. candidate's $a|x| = b$, with at least one of a, b correct) B1

(*b*) Trying to solve either $3x - 1 > 5$ or $3x - 1 < -5$ M1

$3x - 1 > 5 \Rightarrow x > 2$

$3x - 1 < -5 \Rightarrow x < -{}^4/_3$ (both inequalities) A1

Required range: $x < -{}^4/_3$ or $x > 2$ (f.t. one slip) A1

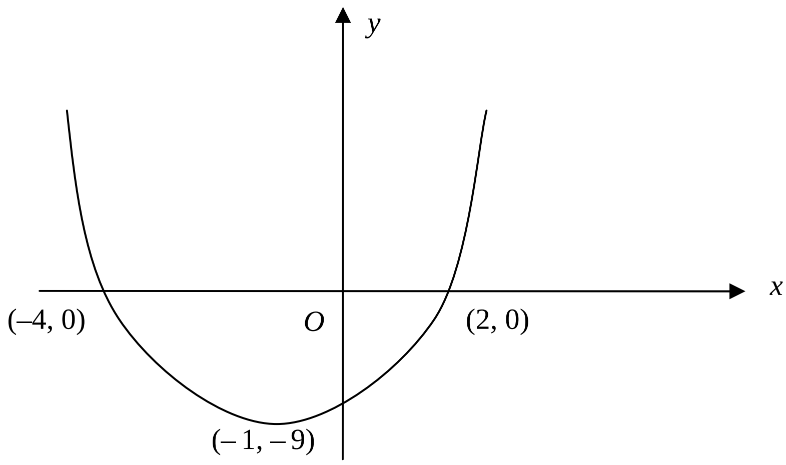
Alternative mark scheme

$(3x-1)^2 > 25$ (forming and trying to solve quadratic) M1

Critical values $x = -{}^4/_3$ and $x = 2$ A1

Required range: $x < -{}^4/_3$ or $x > 2$ (f.t. one slip in critical values) A1

8.



Concave up curve and y-coordinate of minimum = – 9 B1
x-coordinate of minimum = – 1 B1
Both points of intersection with x-axis B1

9. (*a*) $R(f) = [1, \infty)$ B1

(*b*) $y = 4x^2 - 3$ and an attempt to isolate x M1
$4x^2 = y + 3 \Rightarrow x = (\pm)\frac{1}{2}\sqrt{(y + 3)}$ A1
$x = -\frac{1}{2}\sqrt{(y + 3)}$ (f.t. one slip) A1
$f^{-1}(x) = -\frac{1}{2}\sqrt{(x + 3)}$ (f.t. candidate's expression for x) A1
$R(f^{-1}) = (-\infty, -1]$, $D(f^{-1}) = [1, \infty)$
(both intervals, f.t. candidate's $R(f)$) B1

(*c*) (i) $f^{-1}(6) = -{}^3/_2$ (f.t. only for omitted – sign in candidate's correct expression for $f^{-1}(x)$) B1
(ii) Evaluation of $f(k)$, where k is candidate's value for $f^{-1}(6)$ M1
$f(-{}^3/_2) = 6$ (c.a.o.) A1

10. (*a*) $gf(x) = 4[f(x)]^3 + 7$ M1
$gf(x) = 4(e^x)^3 + 7 = 4e^{3x} + 7$ A1

(*b*) $D(gf) = [0, \infty)$ B1
$R(gf) = [11, \infty)$ B1

(*c*) (i) $gf(x) = 18 \Rightarrow 4e^{3x} + 7 = 18 \Rightarrow e^{3x} = {}^{11}/_4 \Rightarrow 3x = \ln({}^{11}/_4)$
(f.t. candidate's gf and allow one algebraic slip) M1
$x = 0{\cdot}337$ (c.a.o.) A1
(ii) Either: e.g. $k = 9$ since $9 \notin R(gf)$ (f.t. candidate's $R(gf)$)
Or: Verification that candidate's choice of k does not yield a value of $x \in D(gf)$ B1