



GCE AS/A level

975/01

MATHEMATICS C3

Pure Mathematics

P.M. WEDNESDAY, 19 January 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_4^6 \frac{1}{3 - \sqrt{x}} dx.$$

Show your working and give your answer correct to three decimal places.

[4]

2. (a) Show, by counter-example, that the statement

$$\sec^2 \theta \equiv 1 - \operatorname{cosec}^2 \theta$$

is false.

[2]

- (b) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$3 \operatorname{cosec}^2 \theta = 11 - 2 \cot \theta.$$

[6]

3. (a) Given that

$$x^4 + 3x^2y - 2y^2 = 15,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y .

[4]

- (b) Given that $x = \ln t$, $y = t^3 - 7t$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t ,

(ii) find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{1}{3}$.

[8]

4. **You may assume** that the equation $6x^4 + 7x - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \frac{3 - 6x_n^4}{7}$$

with $x_0 = 0.4$ can be used to find α . Find and record the values of x_1, x_2, x_3, x_4 .

Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places.

[5]

5. (a) Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(i) $\sqrt{2 + 5x^3}$

(ii) $x^2 \sin 3x$

(iii) $\frac{e^{2x}}{x^4}$

[8]

- (b) By first writing $y = \tan^{-1}x$ as $x = \tan y$, find $\frac{dy}{dx}$ in terms of x .

[4]

6. (a) Find

$$(i) \int \cos 4x \, dx, \quad (ii) \int 5e^{2-3x} \, dx, \quad (iii) \int \frac{3}{(6x-7)^5} \, dx. \quad [6]$$

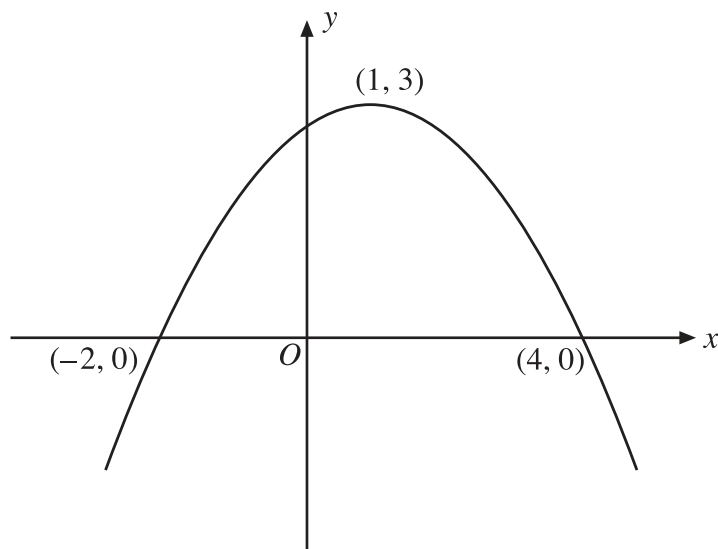
(b) Evaluate $\int_1^4 \frac{9}{2x+5} \, dx$, giving your answer correct to three decimal places. [4]

7. Solve the following.

(a) $5|x| + 1 = 7 - 3|x|$ [2]

(b) $|3x - 1| > 5$ [3]

8. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$ and $(4, 0)$ and has a maximum point at $(1, 3)$.



Sketch the graph of $y = -3f(x + 2)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x -axis. [3]

TURN OVER

9. The function f has domain $(-\infty, -1]$ and is defined by

$$f(x) = 4x^2 - 3.$$

- (a) Write down the range of f . [1]
- (b) Find an expression for $f^{-1}(x)$ and write down the range and domain of f^{-1} . [5]
- (c) (i) Evaluate $f^{-1}(6)$.
- (ii) By carrying out an appropriate calculation involving f , verify that your answer to part (i) is correct. [3]

10. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$\begin{aligned} f(x) &= e^x, \\ g(x) &= 4x^3 + 7. \end{aligned}$$

- (a) Find and simplify an expression for $gf(x)$. [2]
- (b) Find the domain and range of gf . [2]
- (c) (i) Solve the equation $gf(x) = 18$. Give your answer correct to three decimal places.
- (ii) Giving a reason, write down a value for k so that $gf(x) = k$ has no solution. [3]