Mathematics S1 January 2012

Q	Solution	Mark	Notes
1 (a)	P(3 boys) $= \frac{6}{14}\times\frac{5}{13}\times\frac{4}{12}$ or $\binom{6}{3}\div\binom{14}{3}$	M1	
	$= \frac{5}{91}$ (0.055)	A1	
(b)	P(2 boys) $= \frac{6}{14}\times\frac{5}{13}\times\frac{8}{12}\times 3$ or $\binom{6}{2}\times\binom{8}{1}\div\binom{14}{3}$	M1A1	
	$= \frac{30}{91}$		This line need not be seen.
	P(More boys) = Sum of these probs	M1	
	$= \frac{35}{91}$ (5/13,0.385)	A1	FT previous work if first 2 M marks awarded.
2	E(Y) = 2 × 5 + 3 = 13	M1A1	M1 Use of formula, A1 answer.
	Var(X) = 5 si	B1	
	Var (Y) = 4 × 5 = 20	M1A1	M1 Use of formula, A1 answer.
3(a)(i)	$P(X = 7) = \binom{10}{7}\times 0.6^7\times 0.4^3$	M1	Accept 0.3823 – 0.1673 or 0.8327 – 0.6177
	= 0.215	A1	Working must be shown.
(ii)	Use of the fact that if X' denote the number of times Ben wins, X' is B(10,0.4).	M1	
	We require $P(X' \le 4)$	m1	Award m1 for use of adjacent row or column.
	= 0.6331	A1	Working must be shown in (ii).
(b)			Award M1 for summing probs and further 2 marks if correct.
	P(1st win on 4th game) = $0.4\times0.4\times0.4\times0.6$	M1A1	M1 multiplic of relevant probs.
	= 0.0384 (24/625)	A1	
4(a)	P(A∩B) = P(B) ×P(A|B)	M1	
	= 0.06	A1	
(b)	P(A∪B) = P(A) + P(B) – P(A∩B)	M1	
	= 0.54	A1	FT from (a)
(c)	$P(B\|A) = \frac{P(A\cap B)}{P(A)}$	M1	
	= 0.15	A1	FT from (a) except if independence assumed.
5(a)	P(red) $= \frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times 1$	M1A1	M1 Use of Law of Total Prob (Accept tree diagram)
	$= \frac{2}{3}$	A1	Accept Prob = No.of red cards divided by number of cards =6/9
(b)	$P(A\|red) = \frac{1/9}{2/3}$	B1B1	FT denominator from (a) B1 num, B1 denom
	$= \frac{1}{6}$ cao	B1	

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6(a)(i)	$P(X = 5) = \dfrac{e^{-3.6} \times 3.6^5}{5!}$	M1	Working must be shown.
	$= 0.138$	A1	
(ii)	$P(X < 3) = e^{-3.6}\left(1 + 3.6 + \dfrac{3.6^2}{2}\right)$	M1A1	Working must be shown. Award M1 for two correct terms.
	$= 0.303$	A1	
(b)	$P(3 \le X \le 7) = 0.9692 - 0.3027$ or $0.6973 - 0.0308$	B1B1	B1 for each correct prob.
	$= 0.666$ or 0.667 (cao)	B1	
7(a)	$E(X) = 0.1\times1 + 0.1\times2 + 0.2\times3 + 0.2\times4 + 0.4\times5$	M1	M1 Use of $\Sigma x p_x$.
	$= 3.7$	A1	
	$E(X^2) = 0.1\times1 + 0.1\times4 + 0.2\times9 + 0.2\times16 + 0.4\times25$	B1	Need not be seen
	$= 15.5$		
	$\text{Var}(X) = 15.5 - 3.7^2 = 1.81$	M1A1	M1 Use of correct formula for variance.
(b)	$E\left(\dfrac{1}{X^2}\right) = 0.1\times1 + 0.1\times\dfrac{1}{4} + 0.2\times\dfrac{1}{9} + 0.2\times\dfrac{1}{16}$	M1A1	M1 Use of correct formula. A1 completely correct.
	$+ 0.4\times\dfrac{1}{25}$		
	$= 0.176$	A1	
(c)(i)	Possibilities are 1,5; 2,4; 3,3 si	B1	
	$P(\text{Sum} = 6) = 0.1\times0.4\times2 + 0.1\times0.2\times2 + 0.2\times0.2$	M1A1	Award M1A0 if 2s are missing
	$= 0.16$	A1	
(ii)	Possibilities are 1,1; 2,2; 3,3; 4,4; 5,5 si	B1	
	$\text{Prob} = 0.1^2 + 0.1^2 + 0.2^2 + 0.2^2 + 0.4^2$	M1	
	$= 0.26$	A1	
8(a)	We are given that		
	$16p(1 - p) = 2.56$	M1	
	$p^2 - p + 0.16 = 0$	A1	
	Solving by a valid method	M1	
	$p = 0.2$ cao	A1	Award A0 if 0.2 and 0.8 given.
	Accept finding correct solution by inspection.		
(b)	$E(X^2) = \text{Var}(X) + [E(X)]^2$	M1	
	$= 2.56 + 3.2^2$	A1	FT on p for $E(X)$ but not $\text{Var}(X)$.
	$= 12.8$	A1	

Q	Solution	Mark	Notes
9(a)(i)	Using the fact that $F(3) = 1$,	M1	
	$6k = 1$ so $k = 1/6$	A1	
(ii)	$P(X > 2) = 1 - F(2)$	M1	
	$= 2/3$	A1	
(iii)	The median satisfies		
	$\frac{1}{6}\left(m^2 - m\right) = \frac{1}{2}$	M1	
	$m^2 - m - 3 = 0$	A1	
	$m = \frac{1 \pm \sqrt{1+12}}{2} = 2.30$	m1A1	M1 valid attempt to solve.
(b)(i)	$f(x) = F'(x)$	M1	
	$= \frac{2x-1}{6}$	A1	
(ii)	$E(X) = \frac{1}{6}\int_1^3 x(2x-1)\mathrm{d}x$	M1A1	M1 for the integral of $xf(x)$, A1 for completely correct although limits may be left until 2nd line. FT from (b)(i) if M1 awarded there
	$= \frac{1}{6}\left[\frac{2x^3}{3} - \frac{x^2}{2}\right]_1^3$	A1	
	$= 2.22$	A1	