Mathematics C3 January 2013

Solutions and Mark Scheme

Final Version

1.

1	0·211941557		
1·25	0·182137984		
1·5	0·154280773		
1·75	0·128955672		
2	0·106506978	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}25$ — M1

$I \approx \frac{0{\cdot}25}{3} \times \{0{\cdot}211941557 + 0{\cdot}106506978 + 4(0{\cdot}182137984 + 0{\cdot}128955672) + 2(0{\cdot}154280773)\}$

$I \approx 1{\cdot}871384705 \times 0{\cdot}25 \div 3$

$I \approx 0{\cdot}155948725$

$I \approx 0{\cdot}156$ (f.t. one slip) — A1

Note: Answer only with no working earns 0 marks

2. (*a*) (i) e.g. $\theta = 20°$

$\cos^3\theta = 0{\cdot}83$ (choice of θ and one correct evaluation) — B1

$1 - \sin^3\theta = 0{\cdot}96$ (both evaluations correct but different) — B1

(ii) $\theta = 0°$ or $\theta = 90°$ — B1

(*b*) $4(1 + \cot^2\theta) = 9 - 8\cot\theta$. (correct use of $\text{cosec}^2\theta = 1 + \cot^2\theta$) — M1

An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = candidate's coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant — m1

$4\cot^2\theta + 8\cot\theta - 5 = 0 \Rightarrow (2\cot\theta - 1)(2\cot\theta + 5) = 0$

$\Rightarrow \cot\theta = \frac{1}{2}, \cot\theta = -\frac{5}{2}$

$\Rightarrow \tan\theta = 2, \tan\theta = -\frac{2}{5}$ (c.a.o.) — A1

$\theta = 63{\cdot}43°, 243{\cdot}43°$ — B1

$\theta = 158{\cdot}2°, 338{\cdot}2°$ — B1, B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\tan\theta = +, -$, f.t. for 3 marks, $\tan\theta = -, -$, f.t. for 2 marks

$\tan\theta = +, +$, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$\frac{d}{dx}(5x^4y) = 5x^4\frac{dy}{dx} + 20x^3y$ B1

$\frac{d}{dx}(x^3) = 3x^2, \ \frac{d}{dx}(7) = 0$ B1

$\frac{dy}{dx} = \frac{20x^3y + 3x^2}{6y^2 - 5x^4}$ (o.e.) (c.a.o.) B1

(*b*) (i) candidate's *x*-derivative $= 3t^2$ B1

candidate's *y*-derivative $= 4t^3 + 35t^4$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{4t^3 + 35t^4}{3t^2}$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left[\frac{dy}{dx}\right] = \frac{4 + 70t}{3}$ (o.e.) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dx}\right] \div \frac{dx}{dt}$

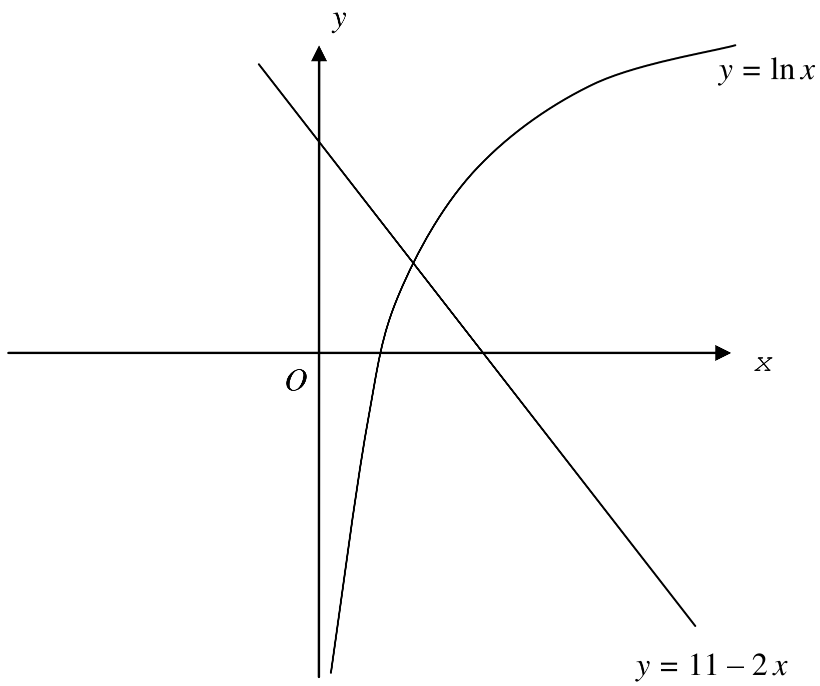
(f.t. candidate's expression for $\frac{dy}{dx}$) M1

$\frac{d^2y}{dx^2} = \frac{4 + 70t}{9t^2}$ (o.e.) A1

(iii) An attempt to solve $t^3 - 5 = 3$ and substitution of answer in candidate's expression for $\frac{d^2y}{dx^2}$ M1

$\frac{d^2y}{dx^2} = 4$ (c.a.o.) A1

4. (*a*)



Correct shape for $y = \ln x$, including the fact that the y-axis is an asymptote at $-\infty$ B1

A straight line with positive intercept and negative gradient intersecting once with $y = \ln x$ in the first quadrant. B1

Equation has one root (c.a.o.) B1

(*b*) $x_0 = 4{\cdot}7$

$x_1 = 4{\cdot}726218746$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 4{\cdot}723437268$

$x_3 = 4{\cdot}723731615$

$x_4 = 4{\cdot}723700458 = 4{\cdot}72370$ (x_4 correct to 5 decimal places) B1

Let $h(x) = \ln x + 2x - 11$

An attempt to check values or signs of $h(x)$ at $x = 4{\cdot}723695$, $x = 4{\cdot}723705$ M1

$h(4{\cdot}723695) = -1{\cdot}87 \times 10^{-5} < 0$, $h(4{\cdot}723705) = 3{\cdot}45 \times 10^{-6} > 0$ A1

Change of sign $\Rightarrow \alpha = 4{\cdot}72370$ correct to five decimal places A1

5. (*a*) (i) $\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times (10x - 3)$ A1

(ii) $\frac{dy}{dx} = \frac{\pm 7}{\sqrt{(1 - (7x)^2)}}$ **or** $\frac{1}{\sqrt{(1 - (7x)^2)}}$ **or** $\frac{7}{\sqrt{(1 - 7x^2)}}$ M1

$\frac{dy}{dx} = \frac{7}{\sqrt{(1 - 49x^2)}}$ A1

(iii) $\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$ M1

$\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$

(either $f(x) = 1/x$ or $g(x) = 3e^{3x}$) A1

$\frac{dy}{dx} = \frac{e^{3x}}{x} + 3e^{3x} \ln x$ (all correct) A1

(*b*) $\frac{d}{dx}(\cot x) = \frac{\sin x \times m \sin x - \cos x \times k \cos x}{\sin^2 x}$ $(m = 1, \mathbf{-1}, k = \mathbf{1}, -1)$ M1

$\frac{d}{dx}(\cot x) = \frac{\sin x \times (-\sin x) - \cos x \times (\cos x)}{\sin^2 x}$ A1

$\frac{d}{dx}(\cot x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$\frac{d}{dx}(\cot x) = \frac{-1}{\sin^2 x} = -\text{cosec}^2 x$ (convincing) A1

6. (*a*) (i) $\int \cos\left(\frac{4x+5}{3}\right) dx = k \times \sin\left(\frac{4x+5}{3}\right) + c$ $(k = 1, {}^4/_3, \mathbf{{}^3/_4}, -{}^3/_4, {}^1/_4)$ M1

$\int \cos\left(\frac{4x+5}{3}\right) dx = \frac{3}{4} \times \sin\left(\frac{4x+5}{3}\right) + c$ A1

(ii) $\int e^{2x+9} dx = k \times e^{2x+9} + c$ $(k = 1, 2, \mathbf{{}^1/_2})$ M1

$\int e^{2x+9} dx = {}^1/_2 \times e^{2x+9} + c$ A1

(iii) $\int \frac{3}{(7-2x)^6} dx = \frac{3}{-5k} \times (7-2x)^{-5} + c$ $(k = 1, 2, \mathbf{-2}, -{}^1/_2)$ M1

$\int \frac{3}{(7-2x)^6} dx = \frac{3}{-5 \times -2} \times (7-2x)^{-5} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \frac{1}{3x-4} dx = k \times \ln|3x-4|$ $(k = 1, 3, \mathbf{{}^1/_3})$ M1

$\int \frac{1}{3x-4} dx = \left[{}^1/_3 \times \ln|3x-4|\right]$ A1

A correct method for substitution of limits 2, 44, in an expression of the form $k \times \ln|3x-4|$ $(k = 1, 3, {}^1/_3)$ m1

$\int_2^{44} \frac{1}{3x-4} dx = \ln 4$ (c.a.o.) A1

7. (*a*) Trying to solve either $3x - 4 > 5$ or $3x - 4 < -5$ M1

$3x - 4 > 5 \Rightarrow x > 3$

$3x - 4 < -5 \Rightarrow x < -{}^1/_3$ (both inequalities) A1

Required range: $x < -{}^1/_3$ or $x > 3$ (f.t. one slip) A1

Alternative mark scheme

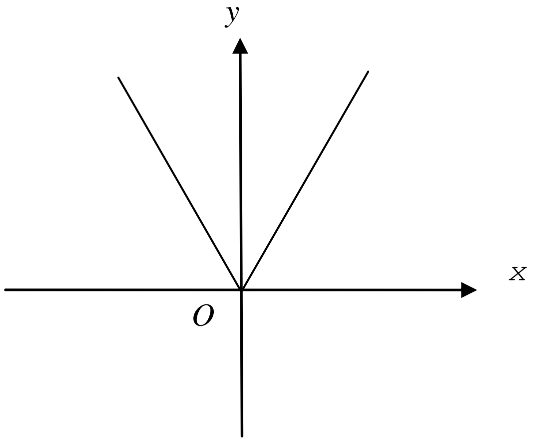
$(3x - 4)^2 > 25$

(squaring both sides, forming and trying to solve quadratic) M1

Critical values $x = -{}^1/_3$ and $x = 3$ A1

Required range: $x < -{}^1/_3$ or $x > 3$ (f.t. one slip in critical values) A1

(*b*) (i)



G1

(ii) $a = -2$ B1

$b = -4$ B1

8. (*a*) $y + 2 = \ln(4x + 5)$ B1

An attempt to express candidate's equation as an exponential equation M1

$x = \frac{(e^{y+2} - 5)}{4}$ (f.t. one slip) A1

$f^{-1}(x) = \frac{(e^{x+2} - 5)}{4}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [-2, \infty)$ B1

9. (*a*) (i) $D(fg) = (0, \infty)$ B1

(ii) $R(fg) = [a, b)$ with

$a = -25$ B1

$b = \infty$ B1

(iii) $fg(x) = (2x - 3)^2 - 25$ B1

(iv) Putting candidate's expression for $fg(x)$ equal to 0 and using a correct method to try and solve the resulting quadratic in x M1

$x = 4$, $x = -1$, (c.a.o.) A1

$x = 4$ (c.a.o.) A1

(*b*) (i)

$$hh(x) = \frac{2 \times \frac{2x + 7}{5x - 2} + 7}{5 \times \frac{2x + 7}{5x - 2} - 2}$$ M1

$$hh(x) = \frac{4x + 14 + 35x - 14}{10x + 35 - 10x + 4}$$

$hh(x) = x$ (convincing) A1

(ii) $h^{-1}(x) = h(x)$ B1