Mathematics S1 January 2013

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1(a)	Use of P(A∪B) + P(A∩B) = P(A) + P(B)	**M1**	
	Use of P(A∩B) = P(A)P(B)	**m1**	
	0.4 + 0.2P(B) = 0.2 + P(B)	**A1**	
	P(B) = 0.25	**A1**	
(b)	EITHER		
	We require $P(A \cap B') + P(A' \cap B)$	**M1**	FT their P(B)
	= 0.2 × (1 – 0.25) + 0.25 × (1 – 0.2)	**A1**	
	= 0.35	**A1**	
	OR		
	We require P(A∪B) – P(A∩B)	**M1**	FT their P(B)
	= 0.4 – 0.2×0.25	**A1**	
	= 0.35	**A1**	
2(a)	E(X) = 3.2, Var(X) = 2.56	**B1B1**	
	E(Y) = 2 × 3.2 + 5 = 11.4 cao	**M1A1**	
	Var(Y) = 4 × 2.56 = 10.24 cao	**M1A1**	
(b)	$Y = 11 \Rightarrow X = 3$	**B1**	
	$P(X = 3) = \binom{16}{3} \times 0.2^3 \times 0.8^{13} = 0.246$	**M1A1**	FT their derived value of X M0 if no working
3(a)	$P(\text{2 red}) = \frac{6}{11} \times \frac{5}{10} \times \frac{5}{9} \times 3$ or $\binom{6}{2}\binom{5}{1} \div \binom{11}{3}$	**M1A1**	
	$= \frac{5}{11}$ (0.455)	**A1**	
(b)	$P(\text{2 green}) = \frac{4}{11} \times \frac{3}{10} \times \frac{7}{9} \times 3$ or $\binom{4}{2}\binom{7}{1} \div \binom{11}{3}$	**M1A1**	
	$= \frac{14}{55}$ (0.255)	**A1**	
	$P(\text{2 the same}) = \frac{5}{11} + \frac{14}{55}$	**M1**	
	$= \frac{39}{55}$ (0.709)	**A1**	FT on their probs

Ques	Solution	Mark	Notes
4(a)(i)	Poisson mean = 6	B1	
	P(4 arrivals) = $e^{-6} \times \frac{6^4}{4!} = 0.134$ cao	M1A1	Accept 0.2851 – 0.1512 or 0.8488 – 0.7149 M0 if no working
(ii)	EITHER		
	P(between 2 and 8) = 0.8472 – 0.0174 or 0.9826 – 0.1528	B1B1	
	= 0.8298 cao	B1	M0 if no working
	OR		
	P(between 2 and 8) = $\sum_{x=2}^{8} e^{-6} \times \frac{6^x}{x!}$	M1	
	= 0.0446 + 0.0892 + 0.1339 + 0.1606 + 0.1606 + 0.1377 + 0.1033	A1	M0 if no working
	= 0.83 cao	A1	
(b)	E(*X*) = 12	B1	
	$E(X^2) = \text{E}(X) + [E(X)]^2 = 156$	M1A1	M1 requires Var(*X*) = E(*X*) FT their mean
5(a)(i)	Let *X* denote the number of seeds producing red flowers so that *X* is B(20,0.7) si	B1	
	P(*X* = 15) = $\binom{20}{15} \times 0.7^{15} \times 0.3^5$	M1	M0 if no working Accept 0.4164 – 0.2375 or 0.7625 – 0.5836
	= 0.179	A1	
(ii)	The number of seeds not producing red flowers, X', is B(20,0.3)	M1	
	We require P(*X* > 12) = P(X' < 8)	m1	
	= 0.7723	A1	
(b)	Number of seeds producing white flowers *Y* is B(150,0.09) ≈ Poi(13.5) si	B1	
	P(*Y* = 10) = $e^{-13.5} \times \frac{13.5^{10}}{10!}$	M1	Do not accept use of interpolation in tables
	= 0.076	A1	M0 if no working

Ques	Solution	Mark	Notes
6(a)	$k(2+3+4+5)=1$ $14k=1$ $k=1/14$	**M1** **A1**	 Must be convincing
(b)	$E(X)=\frac{2}{14}\times1+\frac{3}{14}\times2+\frac{4}{14}\times3+\frac{5}{14}\times4$	**M1**	
	$=\frac{20}{7}$ (2.86)	**A1**	Accept $40k$
	$E(X^2)=\frac{2}{14}\times1+\frac{3}{14}\times4+\frac{4}{14}\times9+\frac{5}{14}\times16$ (65/7)	**B1**	Accept in terms of k
	Var(X) = $65/7-(20/7)^2$ = 1.12 (55/49)	**M1** **A1**	 Numerical value required
(c)	The possibilities are $(x_1,x_2)=(1,2),(2,3),(3,4)$ si	 **B1**	
	Prob = $\frac{2}{14}\times\frac{3}{14}+\frac{3}{14}\times\frac{4}{14}+\frac{4}{14}\times\frac{5}{14}$ = 0.194 (19/98)	**M1A1** **A1**	 Numerical value required
7(a)	P(+) = $0.02\times0.96+0.98\times0.01$ = 0.029	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram)
(b)(i)	P(Disease|+) = $\frac{0.02\times0.96}{0.029}$ = 0.662 (96/145) cao	**B1B1** **B1**	FT denominator from (a) B1 num, B1 denom
(ii)	EITHER P(+) = 0.662 × 0.96 + 0.338 × 0.01 = 0.639	 **M1A1** **A1**	 M1 Use of Law of Total Prob (Accept tree diagram) FT from (b)(i)
	OR P(+) = $\frac{0.02\times0.96^2+0.98\times0.01^2}{0.029}$ = 0.639	 **M1A1** **A1**	 M1 valid attempt to use conditional probability

Ques	Solution	Mark	Notes
8(a)(i)	$P(0.25 \le X \le 0.75) = F(0.75) - F(0.25)$	**M1**	
	$= 0.6875 \quad (11/16)$	**A1**	
(ii)	The median satisfies		
	$2m^2 - m^4 = 0.5$	**B1**	
	$2m^4 - 4m^2 + 1 = 0$		
(iii)	(Root) $= \dfrac{4 \pm \sqrt{16-8}}{4} \quad (= 0.29289..)$	**M1A1**	Condone the omission of the redundant root
	$m = \sqrt{0.29289..} = 0.541$	**M1A1**	
(b)(i)	$f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(2x^2 - x^4)$	**M1**	
	$= 4x - 4x^3$	**A1**	
(ii)	$E\left(\sqrt{X}\right) = \int_0^1 \sqrt{x}(4x - 4x^3)\mathrm{d}x$	**M1A1**	M1 for the integral of $\sqrt{x}f(x)$
	$= \left[4x^{5/2} \times \dfrac{2}{5} - 4x^{9/2} \times \dfrac{2}{9}\right]_0^1$	**A1**	A1 for completely correct although limits may be left until 2nd line.
	$= \dfrac{32}{45} \quad (0.711)$	**A1**	FT their $f(x)$ from (b)(i) if M1 awarded there.