Mathematics C3 January 2014

Solutions and Mark Scheme

Final Version

1. (*a*)

0	0		
$\pi/12$	$0{\cdot}071796769$		
$\pi/6$	$0{\cdot}333333333$		
$\pi/4$	1		
$\pi/3$	3	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/12$ M1

$I \approx \dfrac{\pi/12}{3} \times \{0 + 3 + 4(0{\cdot}071796769 + 1) + 2(0{\cdot}333333333)\}$

$I \approx 7{\cdot}953853742 \times (\pi/12) \div 3$

$I \approx 0{\cdot}69410468$

$I \approx 0{\cdot}6941$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^{\pi/3} \sec^2 x \, dx = \int_0^{\pi/3} 1 \, dx + \int_0^{\pi/3} \tan^2 x \, dx$ M1

$\int_0^{\pi/3} \sec^2 x \, dx = 1{\cdot}7413$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) Choice of x satisfying $75° \leq x < 90°$ and one correct evaluation B1
Both evaluations correct B1

(*b*) $15\,(1 + \cot^2\theta) + 2\cot\theta = 23$
(correct use of $\text{cosec}^2\theta = 1 + \cot^2\theta$) M1
An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = candidate's coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant m1

$15\cot^2\theta + 2\cot\theta - 8 = 0 \Rightarrow (5\cot\theta + 4)(3\cot\theta - 2) = 0$

$\Rightarrow \cot\theta = \frac{2}{3},\ \cot\theta = -\frac{4}{5}$

$\Rightarrow \tan\theta = \frac{3}{2},\ \tan\theta = -\frac{5}{4}$ (c.a.o.) A1

$\theta = 56{\cdot}31°,\ 236{\cdot}31°$ B1

$\theta = 128{\cdot}66°,\ 308{\cdot}66°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta = +, -$, f.t. for 3 marks, $\tan\theta = -, -$, f.t. for 2 marks
$\tan\theta = +, +$, f.t. for 1 mark

3. $\frac{d}{dx}(x^3) = 3x^2$ $\quad \frac{d}{dx}(3) = 0$ B1

$\frac{d}{dx}(-2x^2y) = -2x^2\frac{dy}{dx} - 4xy$ B1

$\frac{d}{dx}(3y^2) = 6y\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{-4}{-14} = \frac{2}{7}$ (c.a.o.) B1

4. (*a*) $\frac{dx}{dt} = 6t^2$ B1

(*b*) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 2 + 12t^2$ B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$ M1

$\frac{d^2y}{dx^2} = \frac{2 + 12t^2}{6t^2}$ (c.a.o.) A1

$\frac{d^2y}{dx^2} = 2 \Rightarrow 2 + 12t^2 = 12t^2$ ($\Rightarrow 2 = 0$) $\Rightarrow$ no such t exists E1

(*c*) Use of $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ M1

$\frac{dy}{dt} = 12t^3 + 24t^5$ (f.t. candidate's expression for $\frac{dx}{dt}$) A1

Use of a valid method of integration to find y m1

$y = 3t^4 + 4t^6$ ($+ c$) (f.t. one error in candidate's $\frac{dy}{dt}$) A1

$y = 3t^4 + 4t^6 + 3$ (c.a.o.) A1

5. $x_0 = 1$

$x_1 = 0{\cdot}612372435$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 0{\cdot}62777008$

$x_3 = 0{\cdot}627136142$

$x_4 = 0{\cdot}627162204 = 0{\cdot}62716$ (x_4 correct to 5 decimal places) B1

Let $h(x) = x^3 + 7x^2 - 3$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}627155$,
$x = 0{\cdot}627165$ M1

$h(0{\cdot}627155) = -6{\cdot}15 \times 10^{-5} < 0$, $h(0{\cdot}627165) = 3{\cdot}81 \times 10^{-5} > 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}62716$ correct to five decimal places A1

6. (*a*) $\frac{dy}{dx} = 10 \times (5x^3 - x)^9 \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = 10(5x^3 - x)^9(15x^2 - 1)$ A1

(*b*) **Either** $\frac{dy}{dx} = \frac{f(x)}{\sqrt{1 - (x^3)^2}}$ (including $f(x) = 1$) **or** $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1 - x^5}}$ M1

$\frac{dy}{dx} = \frac{3x^2}{\sqrt{1 - x^6}}$ A1

(*c*) $\frac{dy}{dx} = x^4 \times f(x) + \ln(2x) \times g(x)$ M1

$\frac{dy}{dx} = x^4 \times f(x) + \ln(2x) \times g(x)$ (either $f(x) = 2 \times \frac{1}{2x}$ or $g(x) = 4x^3$) A1

$\frac{dy}{dx} = x^3 + 4x^3 \ln(2x)$ (all correct) A1

(*d*) $\frac{dy}{dx} = \frac{(2x + 3)^6 \times k \times e^{4x} - e^{4x} \times 6 \times (2x + 3)^5 \times m}{[(2x + 3)^6]^2}$

with either $k = \mathbf{4}$, $m = \mathbf{2}$ or $k = 4$, $m = 1$ or $k = 1$, $m = 2$ M1

$\frac{dy}{dx} = \frac{(2x + 3)^6 \times 4 \times e^{4x} - e^{4x} \times 6 \times (2x + 3)^5 \times 2}{[(2x + 3)^6]^2}$ A1

$\frac{dy}{dx} = \frac{8xe^{4x}}{(2x + 3)^7}$ (correct numerator) A1

(correct denominator) A1

7. (*a*) (i) $\int e^{5x/6}\,dx = k \times e^{5x/6} + c$ $\quad (k = 1, {}^5/_6, \mathbf{{}^6/_5})$ M1

$\int e^{5x/6}\,dx = {}^6/_5 \times e^{5x/6} + c$ A1

(ii) $\int (8x + 1)^{1/3}\,dx = \dfrac{k \times (8x + 1)^{4/3}}{4/3} + c$ $\quad (k = 1, 8, \mathbf{{}^1/_8})$ M1

$\int (8x + 1)^{1/3}\,dx = {}^3/_{32} \times (8x + 1)^{4/3} + c$ A1

(iii) $\int \sin(1 - x/3)\,dx = k \times \cos(1 - x/3) + c$
$(k = -1, \mathbf{3}, -3, {}^1/_3)$ M1

$\int \sin(1 - x/3)\,dx = 3 \times \cos(1 - x/3) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \dfrac{1}{4x - 1}\,dx = k \times \ln(4x - 1)$ $\quad (k = 1, 4, \mathbf{{}^1/_4})$ M1

$\int \dfrac{1}{4x - 1}\,dx = 1/4 \times \ln(4x - 1)$ A1

$k \times [\ln(4a - 1) - \ln 7] = 0{\cdot}284$ $\quad (k = 1, 4, \mathbf{{}^1/_4})$ m1

$\dfrac{4a - 1}{7} = e^{1{\cdot}136}$ (o.e.) (c.a.o.) A1

$a = 5{\cdot}7$ (f.t. $a = 2{\cdot}6$ for $k = 1$ and $a = 2{\cdot}1$ for $k = 4$) A1

8. Trying to solve $3x + 4 = 2(x - 3)$ M1
Trying to solve $3x + 4 = -2(x - 3)$ M1
$x = -10,\ x = 0{\cdot}4$ (c.a.o.) A1

Alternative mark scheme
$(3x + 4)^2 = [2(x - 3)]^2$ (squaring both sides) M1
$5x^2 + 48x - 20 = 0$ (at least two coefficients correct) A1
$x = -10,\ x = 0{\cdot}4$ (c.a.o.) A1

9. (*a*) $y - 1 = \dfrac{2}{\sqrt{3x - 5}}$ B1

An attempt to isolate $3x - 5$ by crossmultiplying and squaring M1

$x = \dfrac{1}{3}\left[5 + \dfrac{4}{(y-1)^2}\right]$ (c.a.o.) A1

$f^{-1}(x) = \dfrac{1}{3}\left[5 + \dfrac{4}{(x-1)^2}\right]$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = (1, 1\cdot5]$ B1 B1

10. (*a*) $g'(x) = \dfrac{4}{(x+1)^2}$ B1

$g'(x) > 0 \Rightarrow g$ is an increasing function B1

(*b*) $R(g) = (0, 4)$ B1 B1

(*c*) $D(fg) = (-\infty, -2)$ B1

$R(fg) = (\sqrt{5}, \sqrt{21})$ (f.t. candidate's $R(g)$) B1

(*d*) (i) $fg(x) = \left(\left[\dfrac{-4}{x+1}\right]^2 + 5\right)^{1/2}$ B1

(ii) Putting expression for $fg(x)$ equal to 3 and squaring both sides M1

$\left[\dfrac{-4}{x+1}\right]^2 = 4$ (o.e.) (c.a.o.) A1

$x = -3, 1$ (two values, f.t. one slip) A1

Rejecting $x = 1$ and thus $x = -3$ (c.a.o.) A1