



GCE AS/A level

0975/01

MATHEMATICS – C3

Pure Mathematics

A.M. WEDNESDAY, 22 January 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \tan^2 x \, dx.$$

Show your working and give your answer correct to four decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \sec^2 x \, dx. \quad [2]$$

2. (a) Show, by counter-example, that the statement

$$\text{'If } x \text{ is an acute angle then } \sin(x + 30^\circ) > \sin x \text{'}$$

is false. [2]

- (b) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$15 \operatorname{cosec}^2 \theta + 2 \cot \theta = 23. \quad [6]$$

3. The curve C is defined by

$$x^3 - 2x^2y + 3y^2 = 3.$$

Find the value of $\frac{dy}{dx}$ at the point $(-2, -1)$. [4]

4. The variables x and y are defined parametrically in terms of the variable t . It is known that

$$x = 2t^3 \text{ and that } \frac{dy}{dx} = 2t + 4t^3.$$

- (a) Find an expression for $\frac{dx}{dt}$ in terms of t . [1]

- (b) Find an expression for $\frac{d^2y}{dx^2}$ in terms of t and hence show there is no value of t for which

$$\frac{d^2y}{dx^2} = 2. \quad [4]$$

- (c) Given that $y = 10$ when $t = 1$, find an expression for y in terms of t . [5]

5. **You may assume** that the equation $x^3 + 7x^2 - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \sqrt{\frac{3}{x_n + 7}}$$

with $x_0 = 1$ can be used to find α . Find and record the values of x_1, x_2, x_3, x_4 .

Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

6. Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(a) $(5x^3 - x)^{10}$ (b) $\sin^{-1}(x^3)$ [2], [2]

(c) $x^4 \ln(2x)$ (d) $\frac{e^{4x}}{(2x + 3)^6}$ [3], [4]

7. (a) Find each of the following, simplifying your answer wherever possible.

(i) $\int e^{\frac{5x}{6}} dx$, (ii) $\int \sqrt[3]{8x + 1} dx$, (iii) $\int \sin\left(1 - \frac{x}{3}\right) dx$. [6]

- (b) Given that $a > 2$, and that

$$\int_2^a \frac{1}{4x - 1} dx = 0.284,$$

find the value of the constant a . Give your answer correct to one decimal place. [5]

8. Solve the equation

$$|3x + 4| = 2|x - 3|. \quad [3]$$

9. The function f has domain $[7, \infty)$ and is defined by

$$f(x) = 1 + \frac{2}{\sqrt{3x - 5}}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]

- (b) Write down the domain of f^{-1} . [2]

TURN OVER

10. The functions f and g have domains $(0, \infty)$ and $(-\infty, -2)$ respectively and are defined by

$$f(x) = \sqrt{x^2 + 5},$$

$$g(x) = \frac{-4}{x+1}.$$

(a) By considering $g'(x)$, show that g is an increasing function. [2]

(b) Write down the range of g . [2]

(c) Write down the domain and range of fg . [2]

(d) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 3. \quad [5]$$

END OF PAPER