Mathematics S1 January 2014

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1(a)(i)	$P(A\cap B) = P(B)P(A \mid B)$ $= 0.08$	**M1** **A1**	Award M1 for using formula
(ii)	$P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$ $= 0.16$	**M1** **A1**	Award M1 for using formula FT their $P(A\cap B)$ unless independence assumed
(b)	Considering any valid expression, eg $P(A\cap B) > 0$, $P(A\|B) > 0$, $P(B\|A) > 0$, $P(A\cup B) < P(A) + P(B)$, the events are not mutually exclusive	**B1**	FT previous work Conclusion must be justified
2(a)	P(1 of each) = $\dfrac{6}{12}\times\dfrac{4}{11}\times\dfrac{2}{10}\times 6$ or $\binom{6}{1}\times\binom{4}{1}\times\binom{2}{1}\div\binom{12}{3}$ $= \dfrac{12}{55}$ (0.218)	**M1A1** **A1**	M1A0 if 6 omitted or incorrect factor used
(b)	P(3 Els) = $\dfrac{6}{12}\times\dfrac{5}{11}\times\dfrac{4}{10}$ or $\binom{6}{3}\div\binom{12}{3}$ $= \dfrac{1}{11}$ (0.091)	**M1** **A1**	
(c)	P(3 Gala) = $\dfrac{4}{12}\times\dfrac{3}{11}\times\dfrac{2}{10}$ or $\binom{4}{3}\div\binom{12}{3}$ $= \dfrac{1}{55}$ (0.018) si P(3 the same) = $\dfrac{1}{11}+\dfrac{1}{55}=\dfrac{6}{55}$ (0.109)	**B1** **M1A1**	FT previous values
3(a)	P(C wins 1st shot) = P(R misses)P(C hits) $= 0.7 \times 0.4$ $= 0.28$	**M1** **A1**	
(b)	P(C wins 2nd shot) = $0.7 \times 0.6 \times 0.7 \times 0.4$ $= 0.42 \times 0.28$ ($k = 0.42$)	**M1** **A1**	
(c)	P(C wins) = $0.28 + 0.42 \times 0.28 + \ldots$ $= \dfrac{0.28}{1-0.42}$ $= 0.483$ (14/29)	**M1** **A1** **A1**	FT their value of k if between 0 and 1

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4(a)(i)	$P(X=6)=\binom{20}{6}\times 0.2^6\times 0.8^{14}=0.109$	**M1A1**	M0 if no working shown
(ii)	Prob $= 0.9900 - 0.0692$ or $0.9308 - 0.0100$ $= 0.921$ cao	**B1B1** **B1**	B0B0B0 if no working shown
(b)	B(200,0.0123) is approx Po(2.46)	**B1**	
	$P(Y=3)=\dfrac{e^{-2.46}\times 2.46^3}{3!}=0.212$	**M1A1**	M0 if no working shown Do not accept use of tables
5(a)	$P(2G)=\dfrac{1}{3}\times 1+\dfrac{1}{3}\times\dfrac{3}{4}\times\dfrac{2}{3}+\dfrac{1}{3}\times\dfrac{2}{4}\times\dfrac{1}{3}$	**M1A3**	M1 Use of Law of Total Prob (Accept tree diagram)
	$=\dfrac{5}{9}$ cao	**A1**	
(b)	$P(A\mid 2G)=\dfrac{1/3}{5/9}$	**B1B1**	FT denominator from (a) B1 num, B1 denom
	$=\dfrac{3}{5}$ cao	**B1**	
6(a)(i)	X is B(10,0.75) si $E(X) = 7.5$, Var(X) = 1.875	**B1** **B1** **B1**	
(ii)	Attempt to evaluate either $P(X = 7)$ or $P(X = 8)$ $P(X = 7) = 0.250$; $P(X = 8) = 0.282$ So try $P(X = 9) = 0.188$ Most likely value = 8	**M1** **A1** **A1** **A1**	Award the final A1 only if the previous A1 was awarded
(b)(i)	$W = 10X - 2(10 - X) = 12X - 20$	**B1**	
(ii)	E(W) $= 12 \times 7.5 - 20 = 70$ Var(W) $= 12^2 \times$ Var(X) $= 270$	**B1** **M1A1**	FT their mean and variance from (a) and FT their derived values of a and b provided that $a \neq 1$ and $b \neq 0$
7(a)	$E(X)=0.1\times 1+0.2\times 2+0.3\times 3+0.1\times 4+0.3\times 5$ $= 3.3$	**M1** **A1**	.
	$E(X^2)=0.1\times 1+0.2\times 4+0.3\times 9+0.1\times 16$ $+0.3\times 25$ (12.7)	**B1**	
	Var(X) $= 12.7 - 3.3^2 = 1.81$	**M1A1**	FT their $E(X^2)$
(b)(i)	The possibilities are (1,1,2); (1,2,1); (2,1,1)	**B1**	
	$P(S=4)=0.1^2\times 0.2\times 3=0.006$	**M1A1**	Award M1 if only one correct possibility given
(ii)	The only extra possibility is (1,1,1) so	**B1**	
	$P(S=3)=0.1^3$ (0.001)	**B1**	
	Therefore $P(S \leq 4) = 0.007$	**B1**	FT from (b)(i) if M1 awarded

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8(a)(i)	Prob = $\frac{e^{-15}\times 15^{12}}{12!}$ or $0.2676 - 0.1848$	**M1**	M0 if no working shown
	$= 0.083$ or $0.8152 - 0.7324$	**A1**	
(ii)	We require $P(X \geq 20)$	**M1**	Award M1A0 for use of adjacent row or column
	$= 1 - 0.8752 = 0.1248$	**A1**	
(b)	(Using tables, the number required is) 25	**M1A1**	Award M1A0 for 24 or 26
9(a)(i)	Using F(2) = 1	**M1**	
	$1 = k(8-2)$	**A1**	
	$k = 1/6$ (convincing)		
(ii)	$P(1.25 \leq X \leq 1.75) = F(1.75) - F(1.25)$	**M1**	
	$= 0.6015\ldots - 0.1171..$ si	**A1**	
	$= 0.484$ (31/64)	**A1**	
(b)(i)	$f(x) = \frac{d}{dx}\left(\frac{x^3 - x}{6}\right)$	**M1**	
	$= \frac{3x^2 - 1}{6}$	**A1**	
(ii)	$E(X) = \int_1^2 x\left(\frac{3x^2 - 1}{6}\right)dx$	**M1A1**	M1 for the integral of $xf(x)$, A1 for completely correct with or without limits FT on their f if previous M1 awarded
	$= \left[\frac{x^4}{8} - \frac{x^2}{12}\right]_1^2$	**A1**	Limits must appear here if not before
	$= 1.625$ cao	**A1**	M0 if no working shown